Lambda Calculus Reduction Strategies

CIS352 — Spring 2023

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Last lecture: reduction **rules** for the lambda calculus

This lecture: reduction **strategies**

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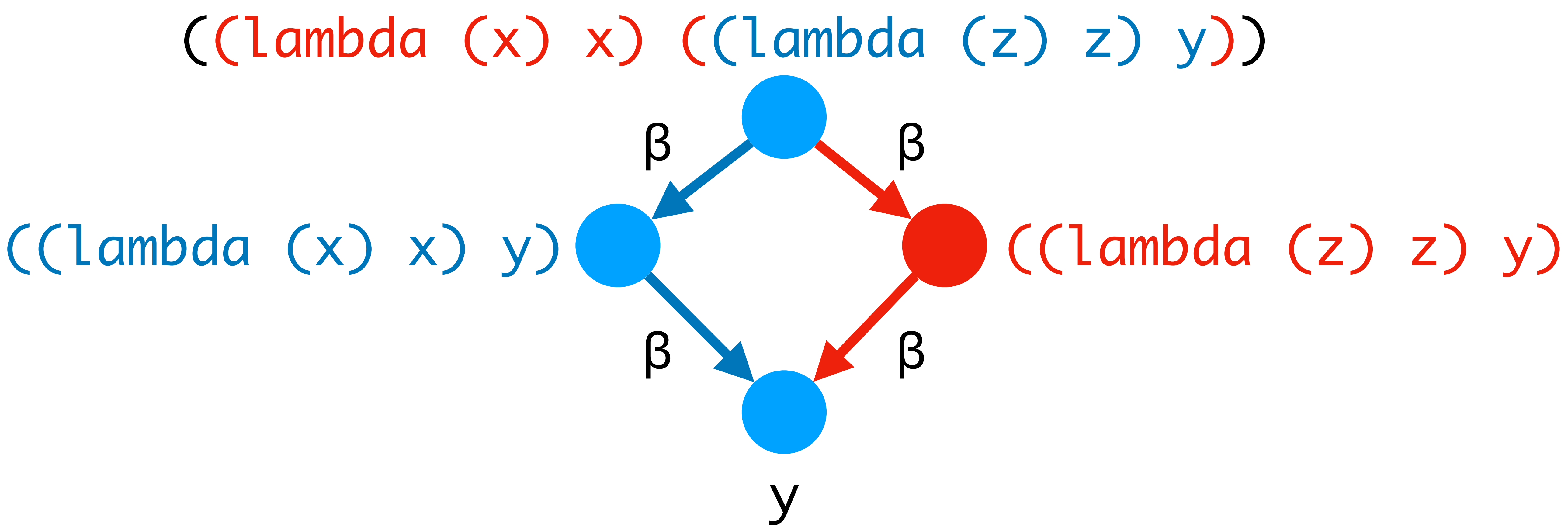

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```
(lambda (x) ((lambda (y) (y y)) (lambda (y) (y y))))
```

We say that lambda expressions are in **Weak Head Normal Form (WHNF)**

Even though a potential redex exists under the lambda, we will not evaluate it (until application)

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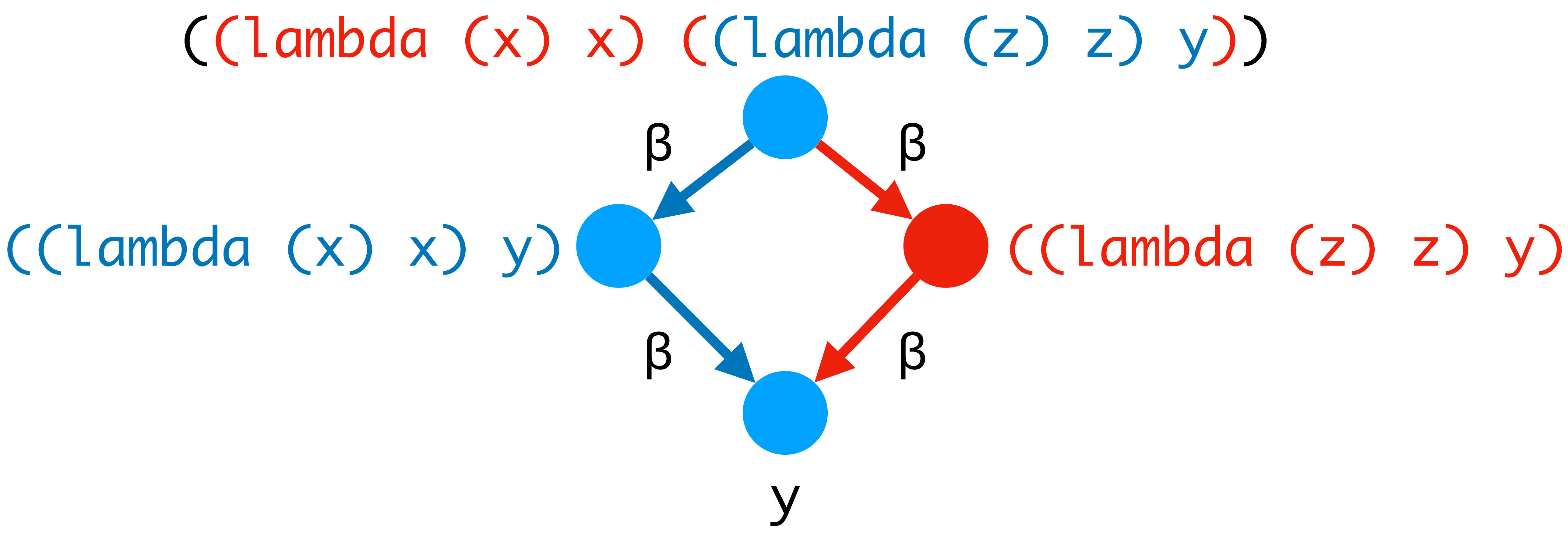
- Call by value, reduce arguments **early** as possible
  - Applicative order (innermost), but **not under lambdas**
- Call by name, reduce arguments **late** as possible
  - Normal order, but **not under lambdas**

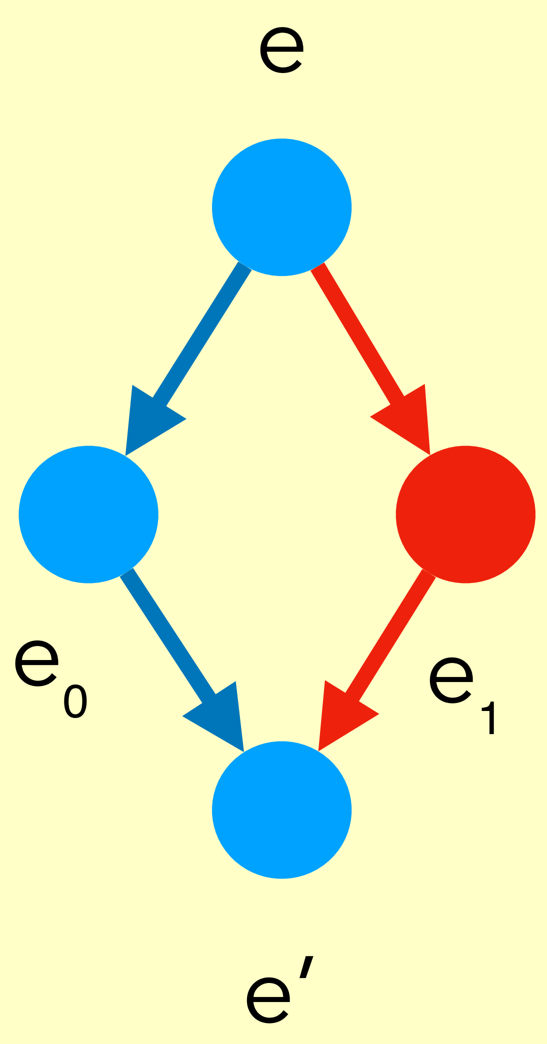
Whenever you get to an application of a lambda, you have a choice:

- Attempt to evaluate argument?
- Perform application immediately



Church-Rosser Theorem

For any expression e,
If e $\rightarrow^*$ e_0 **and** e $\rightarrow^*$ e_1
Then, both e_0 and e_1 step to some **common** term e’



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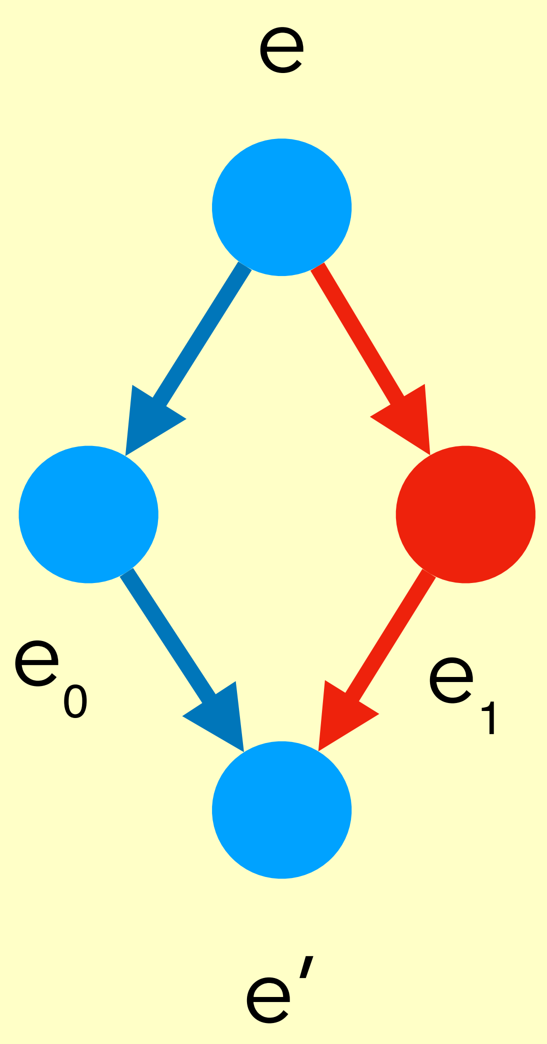
Corollary: all terminating paths result in same normal form!



Give the **reduction sequences** using…

- Call-by-Name
- Call-by-Value

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CBN

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CBV

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Up to alpha equivalence, evaluate this term using:

- Call-by-Name
- Call-by-Value

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((lambda (x) (lambda (y) y))
 ((lambda (x) (x x)) (lambda (x) (x x))))
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Standardization theorem

If an expression can be evaluated to WHNF (i.e., it doesn't loop), then it has a normal-order reduction sequence.

In other words: the lazy semantics is most permissive, in terms of termination.