Textual Reduction

CIS352 — Fall 2022

Kris Micinski

How does the computer evaluate this expression?

```
(* (+ 2 (* 4 6))
   (+ 3 5 7))
```

A C-like language would **compile** the expression

```
int x =
     (2 + 4*6)
  * (3 + 5 + 7);
```

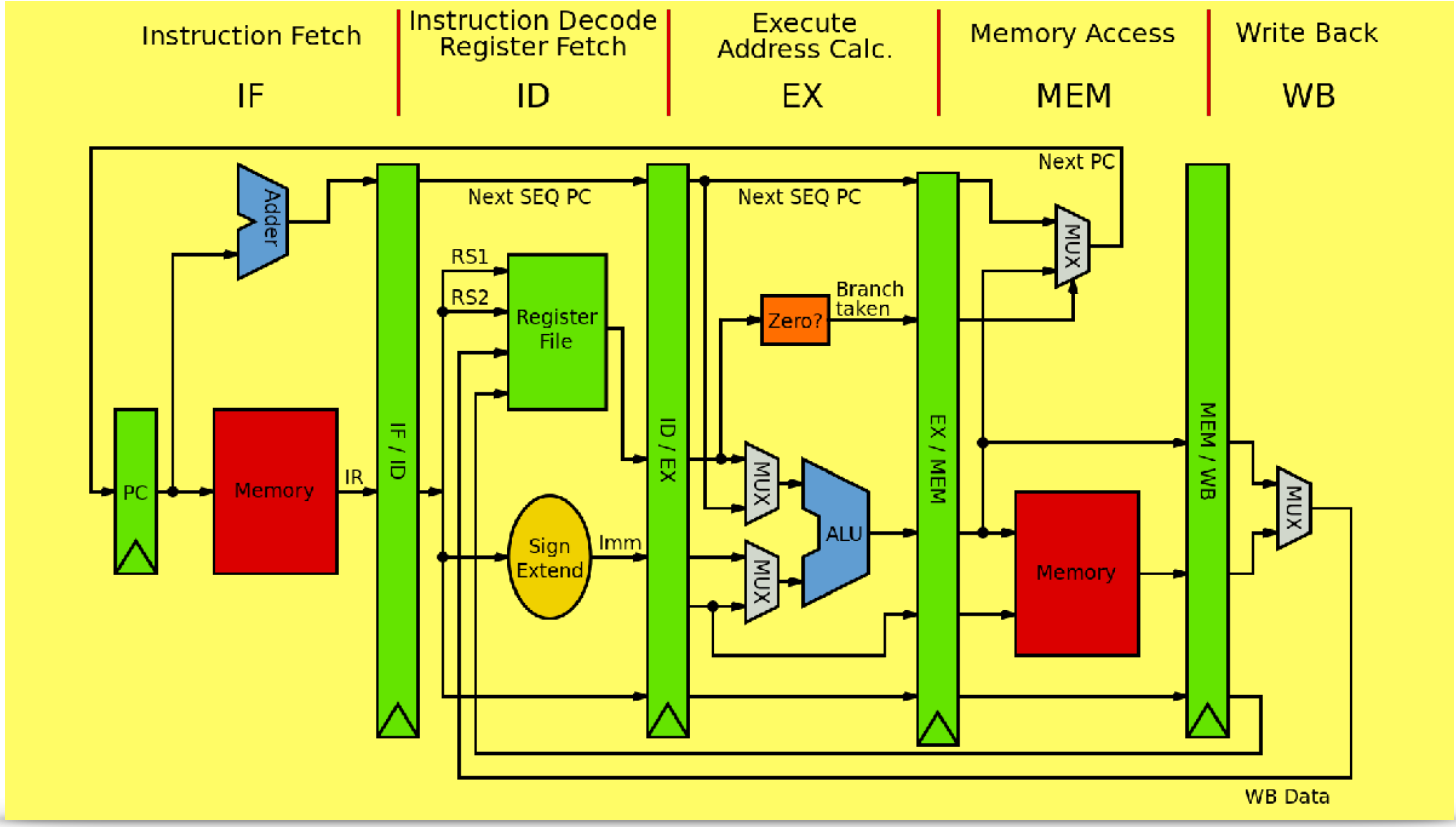
x86-64 clang 13.0.0 (C++, Editor #1, Compiler #1) ✕

x86-64 clang 13.0.0 ▾ | Compiler options...

A ▾ Output... ▾ Filter... ▾ Libraries + Add new... ▾ Add to

```
main:                                   # @main
        push    rbp
        mov     rbp, rsp
        mov     dword ptr [rbp - 4], 0
        mov     dword ptr [rbp - 8], 2
        mov     dword ptr [rbp - 12], 4
        mov     dword ptr [rbp - 16], 6
        mov     eax, dword ptr [rbp - 12]
        imul    eax, dword ptr [rbp - 16]
        mov     dword ptr [rbp - 20], eax
        mov     eax, dword ptr [rbp - 8]
        add     eax, dword ptr [rbp - 20]
        mov     dword ptr [rbp - 24], eax
        mov     dword ptr [rbp - 28], 3
        mov     dword ptr [rbp - 32], 5
        mov     dword ptr [rbp - 36], 7
        mov     eax, dword ptr [rbp - 28]
        add     eax, dword ptr [rbp - 32]
        add     eax, dword ptr [rbp - 36]
        mov     dword ptr [rbp - 40], eax
        mov     eax, dword ptr [rbp - 20]
        imul    eax, dword ptr [rbp - 40]
        mov     dword ptr [rbp - 44], eax
        mov     eax, dword ptr [rbp - 44]
        pop     rbp
```

Computer executes **instructions** on a **clock**



High-level observation:

every computation, in *any* language (running on your processor) is broken down—somehow—into sequences of atomic steps reified as instructions by your processor

A key idea in the course is that evaluation of programs is often broken down into a sequence of small atomic *steps*

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Assembly languages (from your systems course) are a *special case* where the processor's execution makes each *instruction* atomic

Modern microprocessors involve lots of places where atomicity breaks down (cache coherence, etc..) but this is a key abstraction layer in computing

In **high-level languages**, computations/expressions do ***not*** have one-to-one correspondence with the processor's execution.

In fact, it is **impossible** (in *general*) to look at an expression and say how many steps the processor will take to execute an expression

```
;; Some number of steps
(* (+ 2 (* 4 6))
   (+ 3 5 7))
```

Textual reduction is a way of defining the semantics (i.e., meaning) of a program as a series of **progressing steps**, where each step consists of a program (represented **textually**), and a program to which it is "rewritten" (*textually reduced*)

Textual reduction semantics may be defined formally, but in this lecture we will be illustrating them informally

This subexpression is reduced to…

```
(+ (* 3 2) 1)
-> (+ 5 1)
```

This expression, which is a *value*

Values

We often refer to the **values** of a programming language. Intuitively, a value is something that *does not require any additional computation to manifest*

(+ 3 (* (foo 5) 6)) ;; **not** a value
'hello ;; value
15.0 ;; value

In terms of the computation, values are *places where computation stops*

```
(* 3 (+ 4 5))
```

In terms of our intuitive semantics: a builtin function may be applied when each of its arguments is a value

As an aside…

Later, we will see that this construction is inefficient: it means we are doing **at least** O(n) work to (a) identify the redex and (b) then perform a transformation to obtain our result.

Later in the course we will see several improvements to this strategy, e.g., context-and-redex semantics or continuations

At each point in time, we follow a two-step process: identify what can be reduced, and then perform the appropriate reduction

Example reduction sequence

```
(* (* 3 1) (+ 4 5))
-> (* 3 (+ 4 5))
-> (* 3 9)
-> 27 ;; Resulting value
```

Question: in the last slide, why not do **this**?

```
(* (* 3 1) (+ 4 5))
-> (* (* 3 1) 9)
-> (* 3 9)
-> 27 ;; Resulting value
```

Answer: we **could have**! But typically we additionally *constrain* the reductions so that they occur in some predictable order

In most PLs, we process **arguments left-to-right**, then apply builtins when their arguments are values

So far, we have described three rules for reducing arithmetic expressions in a sequence of steps:

- Any number requires no additional work and is a value
- A builtin may be applied when its arguments have been reduced to values
- When we reach a builtin application, we should reduce its arguments from left-to-right

A sequence of reductions (i.e., *steps*) that follow these rules is called a **reduction sequence**

Exercise

Write a reduction sequence for…

```
(+ (* 3 1) (/ 2 2))
```

Exercise

Write a reduction sequence for…

```
(+ (* 3 1) (/ 2 2))
```

```
   (+ (* 3 1) (/ 2 2))
-> (+ 3 (/ 2 2))
-> (+ 3 1)
-> 4 ;; this is a value, computation stops.
```

So far, we have only handled arithmetic. Let's also add **if** and **booleans** to our language. It may also be useful to add builtin comparison operators

IfArith, is a language consisting of numbers, booleans, and arithmetic expressions (plus equality testing), along with if

```
Number ::= 0 | 1 | …
Bool   ::= #t | #f
Value  ::= Number | Bool
Expr   ::= Value
         | (+ expr expr)
         | (* expr expr)
         | (/ expr expr)
         | (= expr expr)
         | (if expr expr expr)
```

We have already covered the highlighted subset

This grammar is in **EBNF (Extended Backus-Naur form)**

```
Number ::= 0 | 1 | …
Bool   ::= #t | #f
Value  ::= Number | Bool
Expr   ::= Value
         | (+ expr expr)
         | (* expr expr)
         | (/ expr expr)
         | (= expr expr)
         | (if expr expr expr)
```

Textual reduction for = happens similarly to + and etc…, except it produces a boolean rather than a number

```
   (= 1 (+ 2 3))
-> (= 1 5)
-> #f
```

```
   (if (= (* 1 (+ 2 3)) 5) 0 1)
-> (if (= (* 1 5) 5) 0 1)
-> (if (= 5 5) 0 1)
-> (if #f 0 1) ;; what next?
```

Q: What happens when you mess up the types?
A: This is one way in which this lecture is inspecific—we have several choices.

For now, we will say that terms that are “ill typed” get ***stuck***, i.e., have no successor states. Later on, we will build type theory to show that well-typed terms *do not get stuck*

```
   (+ (* 1 2) (= 3 4))
-> (+ 2 (= 3 4))
-> (+ 2 #f) <— ☠️☠️☠️; can't make any progress
```

Last, to evaluate an if: first evaluate its guard, then evaluate either the true or false branch based on the guard's value

```
   (if (= 1 (+ 0 1)) (* 2 3) (* 3 1))
-> (if (= 1 1) (* 2 3) (* 3 1))
-> (if #t (* 2 3) (* 3 1))
-> (* 2 3) ;; replace with true branch
-> 6
```

```
   (if (= 1 (+ 1 1)) (* 2 3) (* 3 1))
-> (if (= 1 2) (* 2 3) (* 3 1))
-> (if #f (* 2 3) (* 3 1))
-> (* 3 1) ;; false
-> 6
```

(Informal) Textual Reduction for **IfArith**:

- Any number/bool requires no additional work and is a value
- A builtin (including =) may be applied when its arguments have been reduced to values and are of the right type
- When we reach a builtin application, we should reduce its arguments from left-to-right
- To reduce if, first reduce the guard, then reduce the appropriate branch

A note on state…

In the textual reduction style, we transform a **whole program** to another **whole program**. Thus, the state of the computation is kept in the *current string representing the program*

Looking Forward…

This lecture was an introduction to term-rewriting-style formalisms we will learn later on. **IfArith** is a tiny sub-Turing-complete language we will see again. With the addition of just a single construct, *lambdas* (i.e., functions), we will achieve a Turing-complete language!

The textual reduction style can capture arbitrarily-expressive language features! *But* it is **way** too slow for a real implementation, so we use it as ground truth that is simple to understand. Then we refine to make it fast!

Case Splitting and Lists Intro

CIS352 — Spring 2021

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Cond

- Cond allows multiple guards to be checked
- ```
  (cond [guard0 body0]
        [guard1 body1]
         …
        [else bodyelse]) ;; optional
  ```
- Checks each guard sequentially, evaluates first body

```
(define (foo x)
  (cond [(= x 42) 1]
        [(> x 0)  2]
        [else     3]))
```
```

Exercise

The absolute value of a number x is:

- x is x is greater than 0
- 0 if x = 0
- -x if x is less than 0

Translate this definition into a function using cond

Exercise 

The absolute value of a number x is:

- x is x is greater than 0
- 0 if x = 0
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Translate this definition into a function using cond

```
(define (abs x)
  (cond [(> x 0) x]
        [(= x 0) 0]
        [(< x 0) (- x)]))
```

Exercise

Say we have the following:

```
(cond [g0 b0]
      [g1 b1]
       …
      [else belse])
```

How can we rewrite the above to use only if?

Exercise

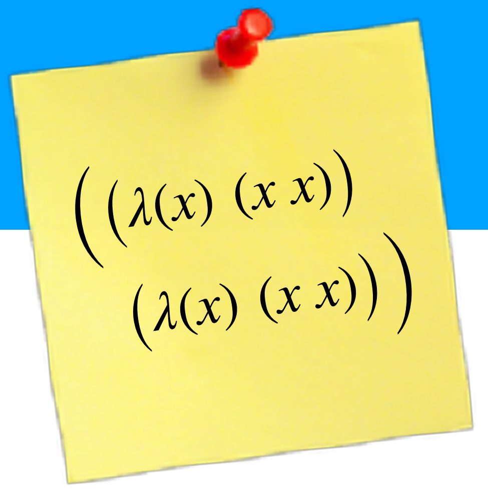
Say we have the following:

```
(cond [g0 b0]
      [g1 b1]
       …
      [else belse])
```

How can we rewrite the above to use only if?

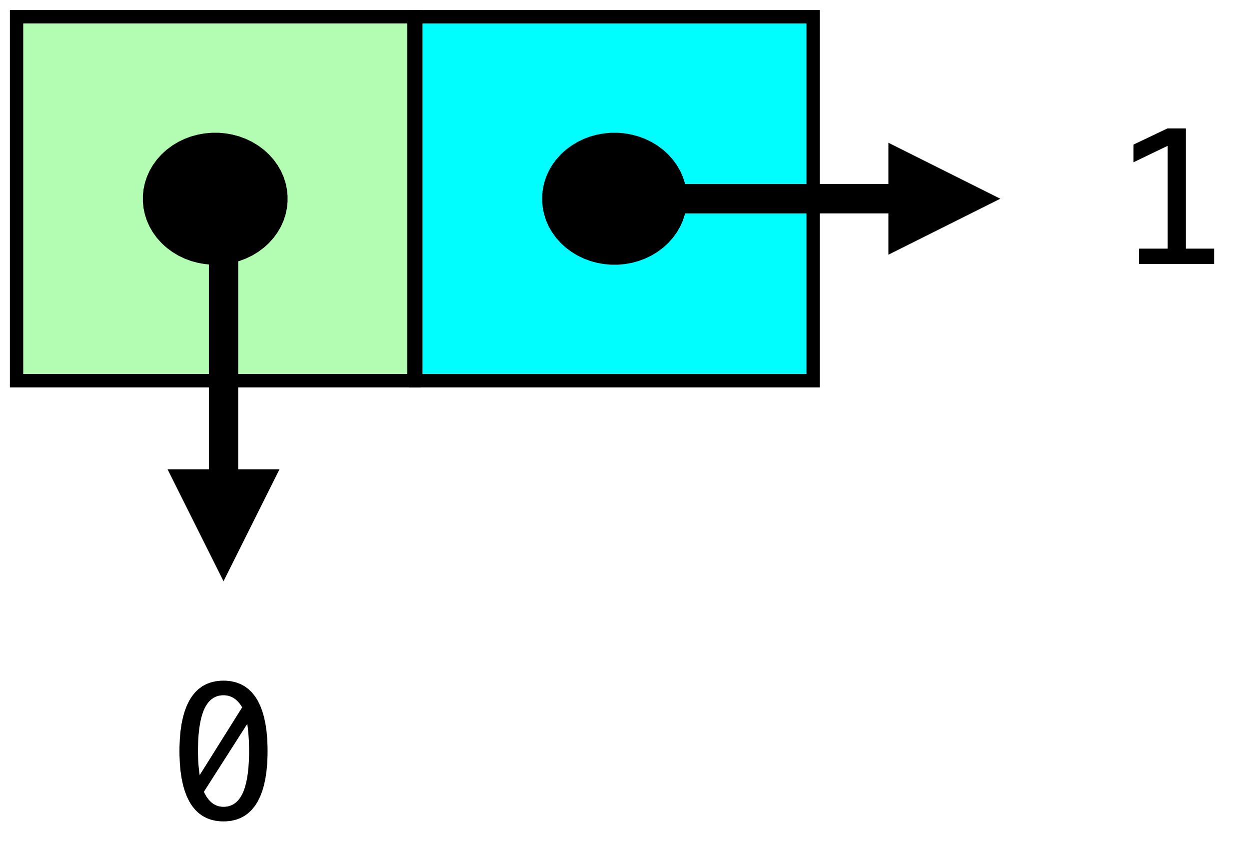
```
(if g0 b0
 (if g1 b1
   …
    (if gn-1 bn-1 belse) …))
```

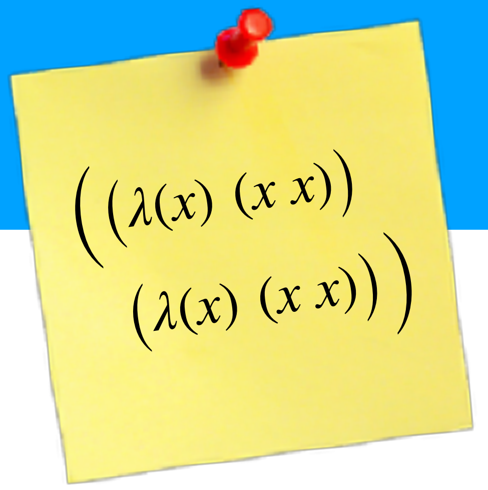
Example



The function **cons** builds a cons cell / pair

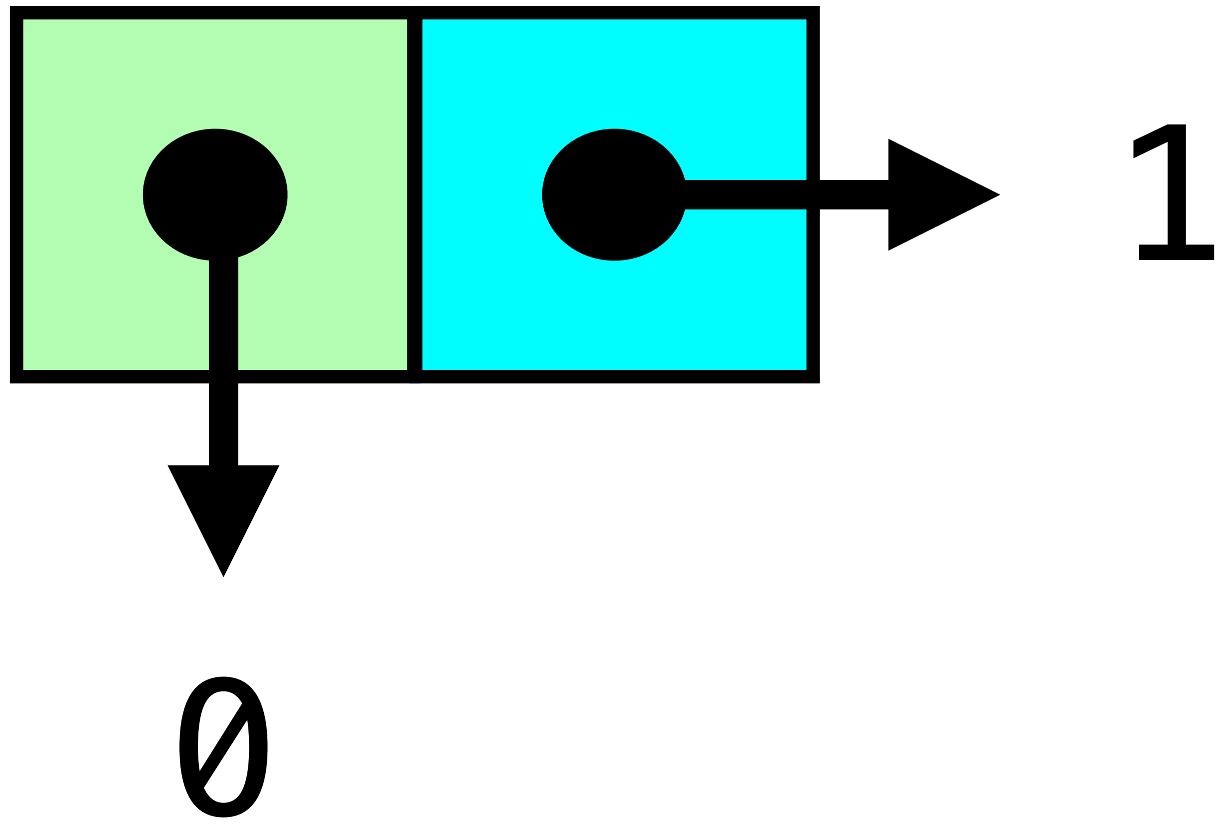
```
(cons 0 1)
```



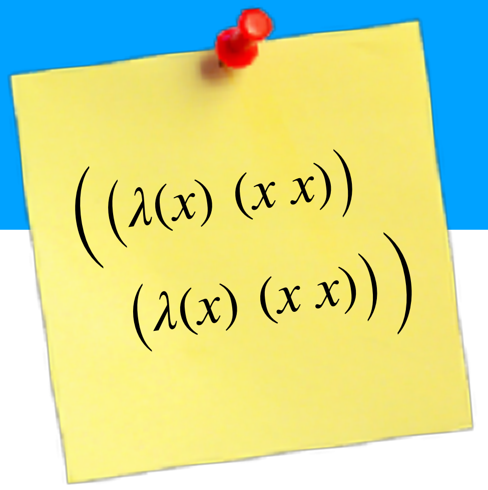


The function **car** gets the left element

`(car (cons 0 1))` is `0`

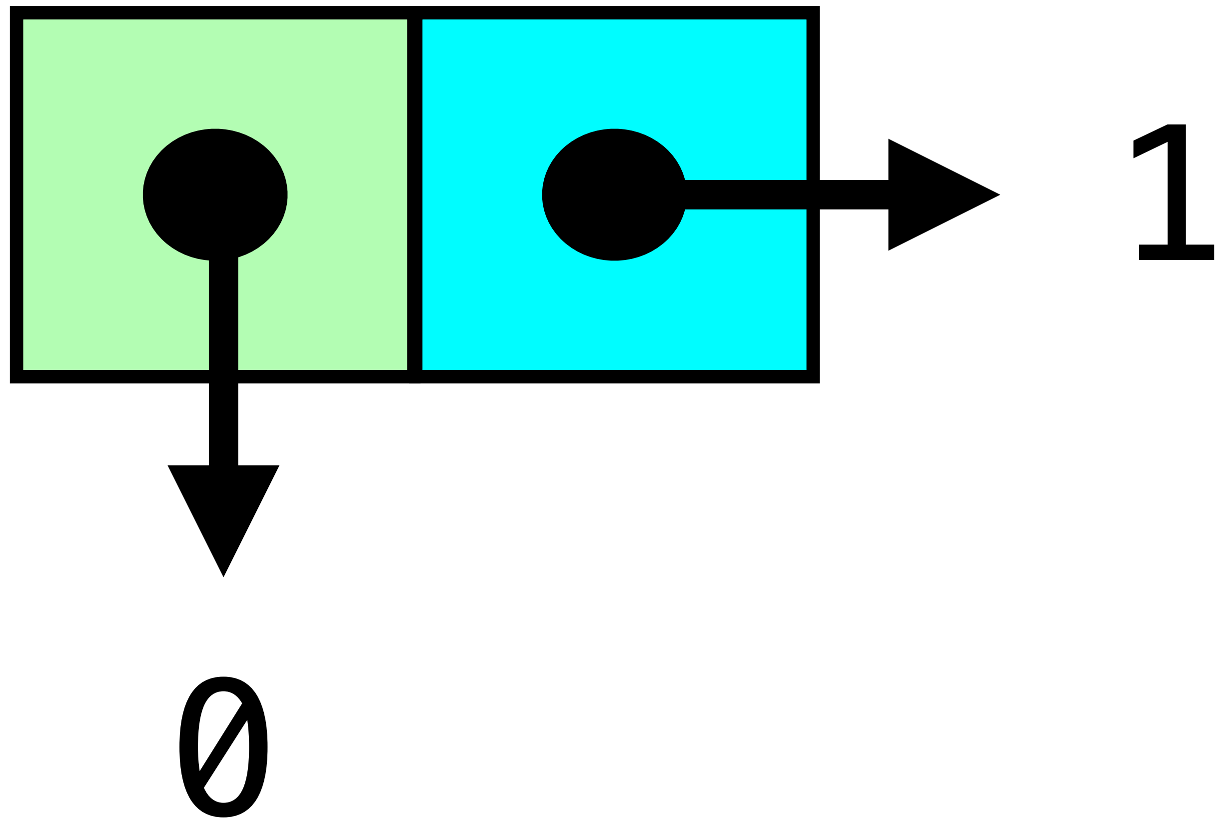


Example

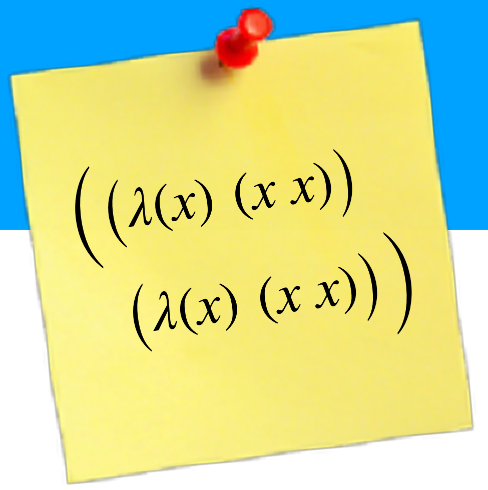


The function **cdr** gets the right element

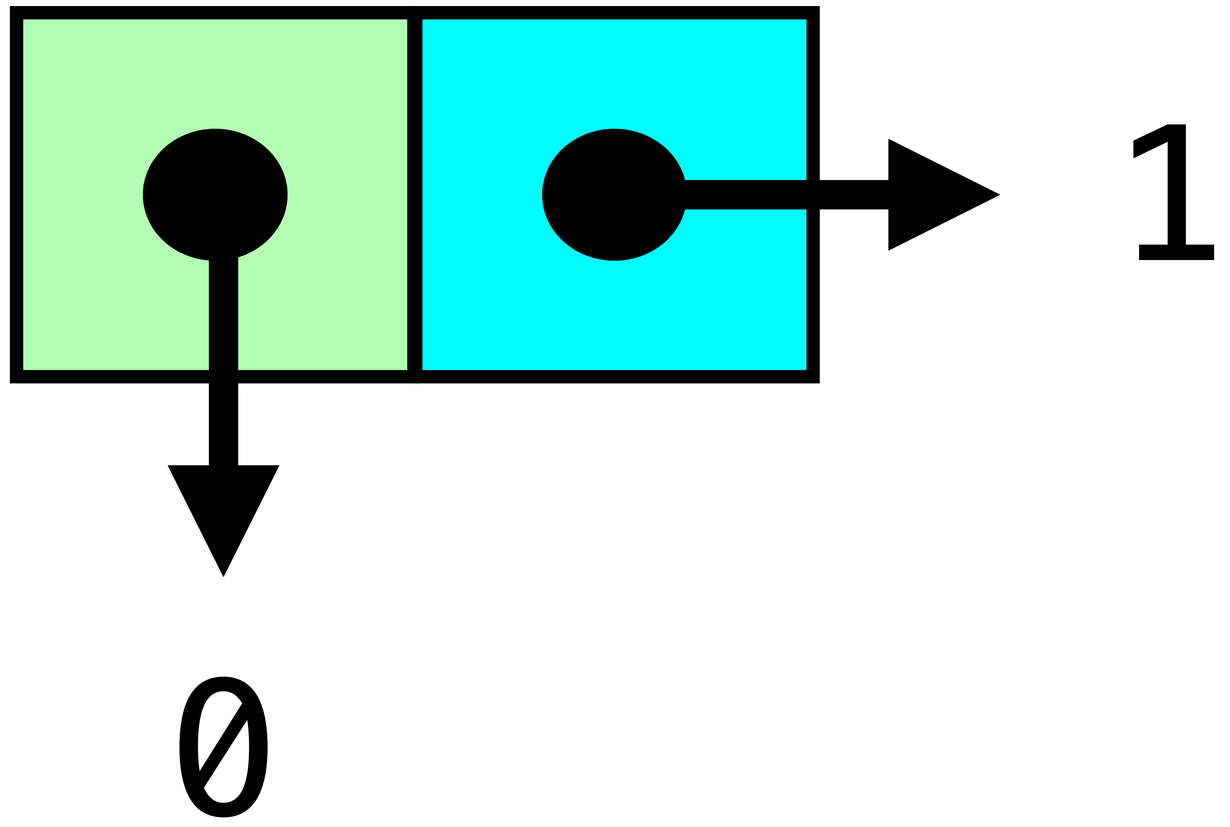
`(cdr (cons 0 1))` is `1`



Example



`(cdr (cons 0 1))` is `1`



The names **car** and **cdr** come from the original implementation of LISP on the IBM 704

Lists

- Racket has **lists**—sequences of cons cells ending w/ `‘()`
- The **empty list** (or “null") is special, `‘()`
- Many ways to build them
  - `(list 1 2 3) ;; Variadic function`
  - `‘(1 2 3)      ;; Datum representation`
- There are **three** operations on lists
  - `empty?` / `null?`
  - `first` / `car`
  - `rest` / `cdr`

Lists continued…

- Using empty?, car, and cdr, we can write many utilities
  - All definable ourselves, also in Racket by default
  - (length l) — Length of l
  - (list-ref l i) — Get ith element of list (0-indexed)
  - (append l0 l1) — Append l1 to the end of l0
  - (reverse l) — Reverse the list
  - (member l x) — Check if x is in l

Exercise

Using cond, write a function that takes a list l and an index x and returns…

- The first element if x = 0
- The second element if x = 1
- The third element if x = 2
- Otherwise return ‘unknown

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