Type Synthesis

CIS352 — Fall 2022

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Type Synthesis

Useful in practice and very interesting!

- If your program typechecks, it is “type correct”
- No need to write annotations everywhere, that’s a pain
- How “really correct” type correctness is depends on the expressivity of the type system:
  - In STLC, it just tells you you get something of the right “shape”
  - In higher-order logics, you can do full verification

Question

Last lecture: Curry-Howard Isomorphism

Curry-Howard tells us that every type system is a logic: does that mean type synthesis is a kind of *proof* synthesis?

Answer: **kind of**, the term is not being synthesized directly, but a proof of the existence of the term is—assuming the type synthesis is *sound*

What is the correct type?

```
(lambda (f) (lambda (x) (if (if-zero? (f x)) 1 0)))
```

Is it:

(a) f = int->int, x = int

(b) f = bool->int, x = bool

(c) f = (int->int)->int, x = int->int

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(a) f = int->int, x = int

(b) f = bool->int, x = bool

(c) f = (int->int)->int, x = int->int

(d) All of the above

Type Variables

```
(lambda (f) (lambda (x) (if (if-zero? (f x)) 1 0)))
```

Lesson:

We can't pick *just one* type. Instead, we need to be able to instantiate f and x whenever a suitable type may be found.

For example, what if we **let-bind** the lambda and use it in two different ways!?

```
(let ([g (lambda (f) (lambda (x) (if (if-zero? (f x)) 1 0)))])
  (+ ((g (lambda (x) x)) 0) ((g (lambda (x) 1)) #f))
```

This usage requires f = nat->nat and x = nat

This usage requires f = bool->nat and x = bool

Generalizations

```
(lambda (f) (lambda (x) (if (if-zero? (f x)) 1 0)))
```

Instead, we can keep a generalized type by using a **type variable**, allowing a good type inference system to derive (for this example, using type var T):

Type of `f = T -> int`

Type of `x = T`

Notice that this system *demands* we must be able to compare T for equality! This is actually *nontrivial* when we add polymorphism, but is simple in STLC (structural equality)

Constraint-Based Typing

The crucial trick to implementing type inference is to use a **constraint-based** approach. In this setting, we *walk over* each subterm in the program and generate a **constraint**

Unannotated lambdas generate new **type variables**, which are later constrained by their usages

Later, we will **solve** these constraints by using a process named **unification**

Building Constraints

```
(define (build-constraints env e)
  (match e
    ;; Literals
    [(? integer? i) (cons `(,i : int) (set))]
    [(? boolean? b) (cons `(,b : bool) (set))]
    ;; Look up a type variable in an environment
    [(? symbol? x) (cons `(,x : ,(hash-ref env x)) (set))]
    ;; Lambda w/o annotation
    [`(lambda (,x) ,e)
     ;; Generate a new type variable using gensym
     ;; gensym creates a unique symbol
     (define T1 (fresh-tyvar))
     (match (build-constraints (hash-set env x T1) e)
       [(cons `(,e+ : ,T2) S)
        (cons `((lambda (,x : ,T1) ,e+) : (,T1 -> ,T2)) S)])]
    ;; Application: constrain input matches, return output
    [`(,e1 ,e2)
     (match (build-constraints env e1)
       [(cons `(,e1+ : ,T1) C1)
        (match (build-constraints env e2)
          [(cons `(,e2+ : ,T2) C2)
           (define X (fresh-tyvar))
           (cons `(((,e1+ : ,T1) (,e2+ : ,T2)) : ,X)
                 (set-union C1 C2 (set `(= ,T1 (,T2 -> ,X)))))])])]
    ;; Type stipulation against t--constrain
    [`(,e : ,t)
     (match (build-constraints env e)
       [(cons `(,e+ : ,T) C)
        (define X (fresh-tyvar))
        (cons `((,e+ : ,T) : ,X) (set-add (set-add C `(= ,X ,T)) `(= ,X ,t)))])]
    ;; If: the guard must evaluate to bool, branches must be
    ;; of equal type.
    [`(if ,e1 ,e2 ,e3)
     (match-define (cons `(,e1+ : ,T1) C1) (build-constraints env e1))
     (match-define (cons `(,e2+ : ,T2) C2) (build-constraints env e2))
     (match-define (cons `(,e3+ : ,T3) C3) (build-constraints env e3))
     (cons `((if (,e1+ : ,T1) (,e2+ : ,T2) (,e3+ : ,T3)) : ,T2)
           (set-union C1 C2 C3 (set `(= ,T1 bool) `(= ,T2 ,T3))))]))
```

Unification

At the end of constraint-building, we have a ton of equality constraints between base types and type variables

```
tv0 = int
ty1 = tv0 -> tv2
tv2 = tv3
tv3 = tv4
```

```
(lambda (x : ty1) …)
```

In this example, what is `ty1`?

Answer: think about constraints and equalities: `ty1` must be `int->int`

Unification

```
;; within the constraint constr, substitute S for T
(define (ty-subst ty X T)
  (match ty
    [(? ty-var? Y) #:when (equal? X Y) T]
    [(? ty-var? Y) Y]
    ['bool 'bool]
    ['int 'int]
    [`(,T0 -> ,T1) `(,(ty-subst T0 X T) -> ,(ty-subst T1 X T))]))

(define (unify constraints)
  ;; Substitute into a constraint
  (define (constr-subst constr S T)
    (match constr
      [`(= ,C0 ,C1) `(= ,(ty-subst C0 S T) ,(ty-subst C1 S T))]))
  ;; Is t an arrow type?
  (define (arrow? t)
    (match t [`(,_ -> ,_) #t] [_ #f]))
  ;; Walk over constraints one at a time
  (define (for-each constraints)
    (match constraints
      ['() (hash)]
      [`((= ,S ,T) . ,rest)
       (cond [(equal? S T)
              (for-each rest)]
             [(and (ty-var? S) (not (set-member? (free-type-vars T) S)))
              (hash-set (unify (map (lambda (constr) (constr-subst constr S T)) rest)) S T)]
             [(and (ty-var? T) (not (set-member? (free-type-vars S) T)))
              (hash-set (unify (map (lambda (constr) (constr-subst constr T S)) rest)) T S)]
             [(and (arrow? S) (arrow? T))
              (match-define `(,S1 -> ,S2) S)
              (match-define `(,T1 -> ,T2) T)
              (unify (cons `(= ,S1 ,T1) (cons `(= ,S2 ,T2) rest)))]
             [else (error "type failure")])])))
```

Why Type Theory?

Why is type synthesis / checking useful?

- Can write **fully-verified** programs.
  - Cons: type systems are esoteric, complicated, academic, etc…
    - Popular languages (Swift, Rust, etc…) are *tending towards more elaborate type systems as they evolve*

- Type synthesis offers me "proofs for free:"
  - "If my program type checks it works" — **not** true in C/C++/…

- Less **mental burden**, like CoPilot (etc… tools), type systems can integrate into IDEs to use synthesis information in guiding programming
  - In some ways, this reflects the logical statements underlying the type system's design (Curry Howard)

PL Research @ SU

Our group: build the world’s fastest fixpoint solvers

Slog: data-parallel deductive logic (Horn-SAT)

- Scaled control-flow analysis up to 1000 cores of the Theta supercomputer

Ascent:

- Programming with **lattices**, parallelization of declarative analytics on large unified-memory machines
- Macro-embedded language in **Rust**
- Parallelization using Rayon

Fast Horn-SAT

Lots of applications use restricted finite-domain propositional logic:
- Transitive closure, triangle counting, k-clique, …

```
[(path x y) <— (edge x y)] ;; Initial step
[(path x z) <— (path x y) (edge y z)] ;; # Inductive rule
```

- We can do programs like these at the **highest scale currently known**
  - Transitive closure: thousands of cores on Theta, graphs w/ billions of edges

Also forms the basis for **program analysis**: on-the-fly reachability of a program’s control-flow graph:
- Scalable

Fast transitive closure at scale

Graph Properties			Time (s) at Process Count				
Name	Edges	|TC|	System	15	30	60	120
FB-MEDIA	206k	96,652,228	SLOG	62	40	21	**18**
			Soufflé	35	33	34	37
			Radlog	254	295	340	164
RING10000	10k	100,020,001	SLOG	363	218	177	**115**
			Soufflé	149	143	140	141
			Radlog	464	646	852	1292
SUITESPARSE	412k	3,354,219,810	SLOG	–	1,593	908	**671**
			Soufflé	1,417	1,349	1,306	1,282
			Radlog	–	–	–	–

Data-Parallel Structured Deduction

New language extending Datalog to S-expressions:
- All structures are deduplicated via a hash value
- Distributed through a cluster via that hash
- Data efficiently indexed to support operations on structures

Allows rich programming in a style that looks much closer to natural deduction

All programs compile to **data-parallel relational algebra kernels** implemented via all-to-all communication on top of MPI

Allows scaling structured logic programming up to hundreds/thousands of threads

Examples in Slog

```
; ref
(interp ?(clo (ref x) env)
        {interp {env-map env x}})
; lam
(interp ?(clo (lam x Eb) env)
        (clo (lam x Eb) env))
; app
[(interp !(clo Ef env)
         (clo (lam x Eb) env'))
 (= env'' (ext-env env' x (clo Ea env)))
 (interp !(clo Eb env'') v)
 -->
 (interp ?(clo (app Ef Ea) env) v)]
```

```
; ref
(interp ?(clo (ref x) env)
        {env-map env x})
; lam
(interp ?(clo (lam x Eb) env)
        (clo (lam x Eb) env))
; app
[(interp !(clo Ef env)
         (clo (lam x Eb) env'))
 (interp !(clo Ea env) Eav)
 (interp !(clo Eb (ext-env env' x Eav)) v)
 -->
 (interp ?(clo (app Ef Ea) env) v)]
```

Fig. 5. Two CE (closure-creating) interpreters in Slog; for CBN eval. (left) and CBV eval. (right).

Control-Flow Analysis of the λ-calculus in Slog

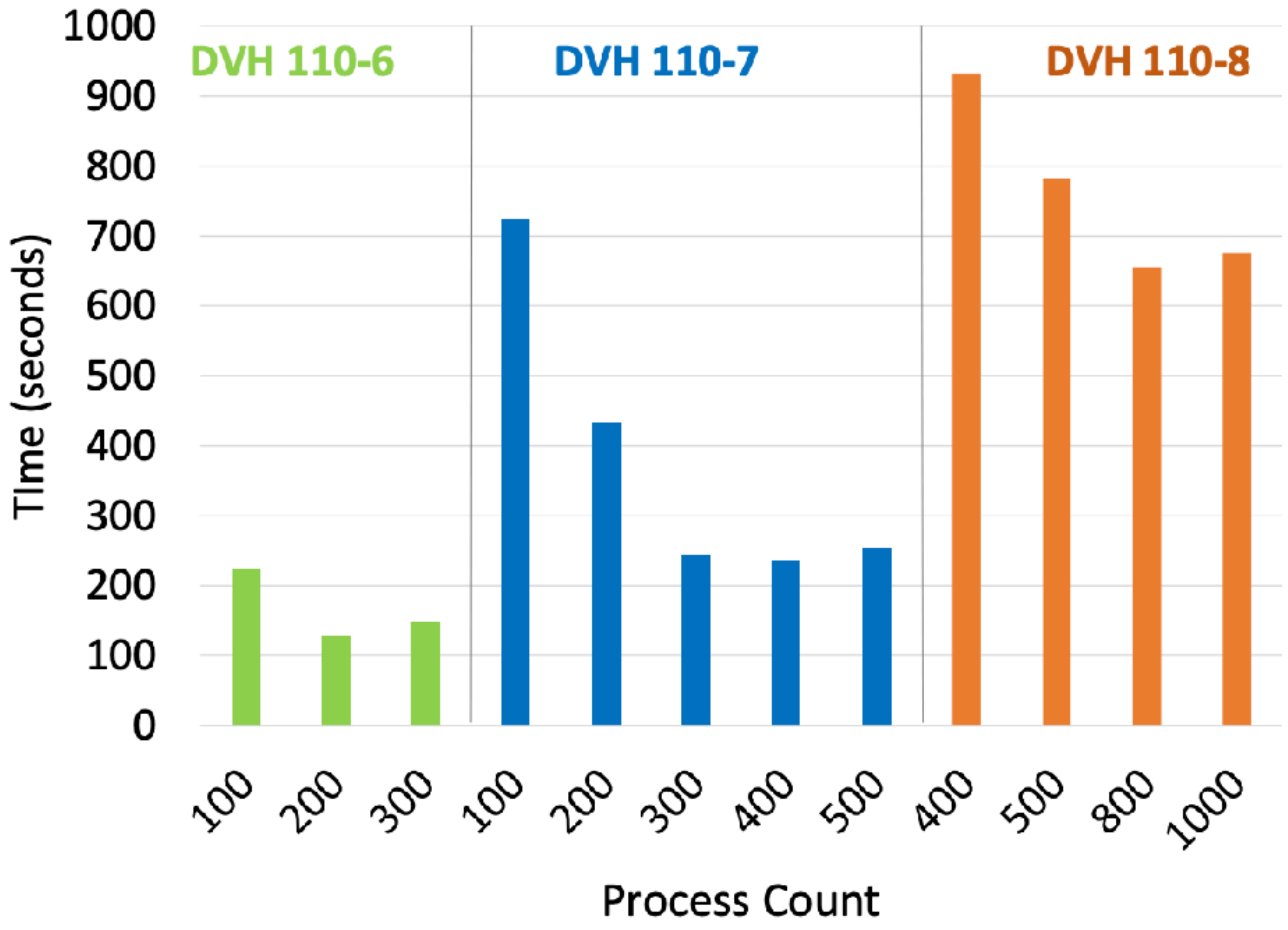
```
;; Eval states
[(eval (ref x) k c)
 -->
 (ret {store (addr x c)} k)]
[(eval (lam x body) k c)
 -->
 (ret (clo (lam x body) c) k)]
[(eval (app ef ea) k c)
 -->
 (eval ef (ar-k ea (app ef ea) c k) c)]


;; Ret states
[(ret vf (ar-k ea call c k))
 -->
 (eval ea (fn-k vf call c k) c)]
[(ret va (fn-k vf call c k))
 -->
 (apply call vf va k c)]
[(ret v (kaddr e c))
 (store (kaddr e c) k)
 -->
 (ret v k)]
;; Apply states
[(apply call (clo (lam x Eb) _) va k c)
 -->
 (eval Eb (kaddr Eb c') c')
 (store (kaddr Eb c') k)
 (store (addr x c') va)
 (= c' {tick !(do-tick call c)})]
; Propagate free vars
[(free y (lam x body))
 (apply call (clo (lam x body) clam) _ _ c)
 -->
 (store (addr y {tick !(do-tick call c)})
        {store (addr y clam)})]
```

CFA of λ-calculus vs. Souffle

	Term Sz.	Iters	Cf. Pts	Sto. Sz.	8 Processes		64 Processes	
					Slog	Soufflé	Slog	Soufflé
3-k-CFA	8	1,193	98,114	23,413	00:01	01:07	0:02	00:15
	9	1,312	371,010	79,861	00:02	14:47	0:03	02:56
	10	1,431	1,441,090	291,317	00:06	⏱	0:05	45:49
	11	1,550	5,678,402	1,107,957	00:27	⏱	0:16	⏱
	12	1,669	22,541,634	4,315,125	02:14	⏱	1:07	⏱
	13	1,788	89,822,530	17,022,965	12:17	⏱	5:08	⏱

Strong Scaling on Theta



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