Racket and FP
Break up into pairs: find someone that has Dr. Racket

(Yes, you have to participate..)
Kris talks about failure
Racket

- Dynamically typed: variables are untyped, values typed

- Functional: Racket emphasizes functional style
  
  - Immutability—Requires automatic memory management

- Imperative: Racket allows values to be strongly-updated, and is thus “impure” as functional languages go
  
  - Often discouraged

- Language-Oriented: Racket is really a language toolkit
A brief tour of history…
We wanted a language that allowed **symbolic manipulation**
List of either **atoms** or **S-expressions**

(this (is an) (s) expression)
List of either atoms or S-expressions

(this (is an) (s) expression)
List of either atoms or S-expressions

((this (is an) (s) expression))
List of either atoms or S-expressions

(this (is an) (s) expression)
List of either atoms or S-expressions

(this (is an) (s) expression)

also an S-expression
SIMPLE EXPRESSION FOR
(CAPPELLD '(A B C) '(D E F))

APPEND =
(LAMBDA (X Y)
  (IF (ATOM X) Y
   (CONS (CAR X)
     (APPEND (CDR X) Y)))))
So how do we write programs in this?
Calls `function` with arguments `arg₀, arg₁, etc…`
Two examples with + and -
Quiz problems: + and -

Calculate $(1 + (2 - 3)) - 4$
Introduce if, and, or
(if #t 1 2)
(if (equal? 2 3) 1 2)
(if (< 3 4) 1 2)
(if (and (or #t) #t) 1 2)
Notice: there is no “return” value

In functional programming, every single expression implicitly returns its resulting value
(and #t #f)

(or #t ...)

Always true, even if … doesn’t terminate!
(define (factorial x)
  (if (equal? x 0)
      1
      (* (factorial (- x 1)) x)))
(define (factorial x)
  (if (equal? x 0)
      1
      (* (factorial (- x 1)) x)))

- Everything in parenthesis
- Prefix notation
- No variable assignment
- Recursion instead of loops
- No types
- No return
Quiz

• Compute the factorial of 5
• Compute the factorial of 20
• Compute the factorial of 20000
Quiz

• Define the fibonacci function:
  • Use if, equal?, -

• \( \text{fib}(0) = 1 \)

• \( \text{fib}(1) = 1 \)

• \( \text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2) \)
Introduce cond
Introduce cond

(cond
  (\[= x 1\] 1)
  (\[= x 2\] 2)
  (else 3))
Introduce cond

(cond
  (\[= \ x \ 1\] 1)
  (\[= \ x \ 2\] 2)
  (else 3))

Any number of conditional “clauses”
Introduce cond

\[
\text{(cond}
\begin{align*}
&([= \ x \ 1] \ 1) \\
&([= \ x \ 2] \ 2) \\
&(\text{else} \ 3))
\end{align*}
\]

Potentially an “else” clause
Introduce cond

\[(\text{cond})\]
\[
\quad (\text{\texttt{[=} x 1]} \ 1)
\]
\[
\quad (\text{\texttt{[=} x 2]} \ 2)
\]
\[
\quad (\text{\texttt{else}} \ 3))
\]

\text{cond} \text{ checks each clause and executes the body of the first one that matches}
If you get stuck, use the debugger...!
Racket is *dynamically typed*
(define (fib-again x)
  (cond
   [(< x 0) (raise 'lessthanzero)]
   [(eq? 0 x) 1]
   [(eq? 1 x) 1]
   [else 0])))
Define max

- cond
- <
- >
- equal?
Most Racket data is based on lists

'(1 2 3)
Most Racket data is based on **lists**


\[
\begin{align*}
\text{'(1 2 3)} & \quad \rightarrow \quad \text{'(1 2 3)} \\
\text{(first '}(1 2 3)) & \quad \rightarrow \quad 1 \\
\text{(rest '}(1 2 3)) & \quad \rightarrow \quad \text{'(2 3)} \\
\text{(rest '}(3)) & \quad \rightarrow \quad \text{'()} \\
\end{align*}
\]
Can use `empty?` to check

`(empty? '())

`(empty? '(1 2))

Pronounced “empty-huh?”
Define max-of-list

- empty?
- first
- rest
- length?
Can create local variables with `let`

```
(let ([x 2]
      [y 3])
  (+ x y))
```

“Let x be 2 and y be 3 inside the expression…”
Quiz

Define \(\text{distance} \ x_1 \ y_1 \ x_2 \ y_2\)

Use \(\text{sqrt}\)

Use \(\text{let}\) at least once
You can create **anonymous** functions with lambda

\[
\text{(lambda (x) (- x))}
\]

\[
\text{(lambda (str) (string-ref str 0))}
\]

\[
(((\text{lambda (x) (* x x) 3)})
\]

\[
(\text{define f (lambda (x) (* 2 x))))
\]

\[
(f 3)
\]
(let ([x 1])
  (+ x 1))

Rewrite this in terms of lambda!
\[(\text{let } ([x \ 1]) \ (\text{+} \ x \ 1)) \rightarrow \ ((\text{lambda} \ (x) \ (\text{+} \ x \ 1))) \ 1)\]

Let is \(\lambda\)
(let* ([x 1]
       [y (+ x 1)])
  (list y x))
Lots of other things are \( \lambda \) too...

\[
\text{(define } (f \ x) \ x) \\
\text{shorthand for...}
\]

\[
\text{(define } f \ (\text{lambda } (x) x))
\]
(define (f x) x)

(define (f x y) x)

(define f (lambda (x) x))

(define f (lambda (x y) x))
Here’s what most confused me…

```plaintext
> (lambda x x)  
#<procedure>  
> (lambda (x) x)  
#<procedure>  
> (lambda (x) x) 1  
#<procedure>  
1  
> (((lambda (x) x) 1) 1 1  
> (((lambda x x) 1)  
'(1)  
> |  
```
Define hyphenate

> (hyphenate '("Kristopher" "Kyle" "Micinski"))
"Kristopher-Kyle-Micinski"

(Use string-append)
Using higher order functions…
If you give me a function, I can use it

\[
  \text{(define twice}
  \text{(lambda (f)}
    \text{(lambda (x)}
      \text{(f (f x))})
  \text{)())}
\]

Challenge: figure out how I would use twice to add 2 to 2

Use Racket’s add1 function

\[
  \text{(add1 (add1 2))}
\]
All the forms we covered today:
Define, let, lambda, cond, if
Data Structures via Lists
LISP is over half a century old and it still has this perfect, timeless air about it.

I wonder if the cycles will continue forever.

A few coders from each new generation re-discovering the LISP arts.

These are your father’s parentheses.

Elegant weapons for a more... civilized age.
In today’s class, we’re going to build all data from three things…
The first is *atoms*

These are the *primitive things* in the language
symbol 1
These are like “int” and “char” in C++
The second is the empty list
The last is cons
Cons is a function that takes two values and makes a pair
That pair is represented as a **cons cell**
(cons 1 2)
CONS is the natural constructor of the language
I use two strange words to refer to the elements of this cons cell
“car”
Because car and cdr break apart what I build with cons, I call them my **destructors**
And that’s all
And that’s all

Atoms

Empty list

cons

(car (cons ‘sym 23))

car/cdr

‘sym 23

‘()

‘\c

(cons ‘sym 23)
Using just this, I can make a list
Using just this, I can make a list

(And everything else in the world, but we’ll get back to that…)
If I want to make the list containing 2 I do this
(cons 2 '(())

2

'()
When I do this, Racket prints it out as a list
The way to read this is

“The list containing 2, followed by the empty list.”
Just as I can build lists of a single element, I can build larger lists from smaller lists…
And I do that by stuffing lists inside other lists…
(cons 2 '(()))
(cons 3 (cons 2 '()))
Racket will print this out as
'(3 2)
Of course, I probably need at least numbers as primitives right?
To get the head of a list, I use `car`
(cons 3 (cons 2 '()))
(car
(cons 3 (cons 2 '()))))
(cdr
  (cons 3 (cons 2 '()))
)
So now how would I get the second element?
(cdr
  (cons 3 (cons 2 '())))

3 3 2 '()
(car
  (cdr
    (cons 3 (cons 2 ‘(())()))))
Racket abbreviates

\[(\text{cons } 1 \ (\text{cons } 2 \ (\text{cons}...\ (\text{cons } n \ ‘()\)...)\)])\]

as...

\[‘(1 \ 2 \ ... \ n)\]
If I wanted to write out lists, I could do so using

\[(\text{cons} \ 1 \ (\text{cons} \ 2 \ \ldots))\]
How do I get the nth element of a list?
(define (nth list n)
  (if (= 0 n)
      (car list)
      (nth (cdr list) (- n 1)))))
Now, write \((\text{map } f \ l)\)
Writing lists would get quite laborious
Instead, I can use the primitive function \texttt{list}
(list 1 2 'serpico)

'(1 2 serpico)
Oh, and actually I can use this to represent trees too
How would I build this?
(define empty-tree 'empty-tree)

(define (make-leaf num) num)

(define (make-tree left right)
  (cons left right))
You define (left-subtree tree)
(define (least-element tree)
  (if (number? tree)
      tree
      (least-element (left-subtree tree))))
But surely I need things like numbers right?
It turns out, you could build those using just cons, car, cdr, if, =, and ‘()
Define the number \( n \) as …

\[ (\ ) \]
\[ ( ( ) ) \]
\[ ( ( ) ( ) ) \]

…
(define (weird-plus i j)
  (if (equal? i '())
      j
      (weird-plus (cdr i)
                  (cons '() j)))))
(weird-plus '(() () '(() ()))
 '(() () () () )
It turns out, if I’m clever, we can even get rid of \textbf{if} and \textbf{equal}

(Though we shall not do so here..)
I can build my own datatypes in this manner
I usually write **constructor** functions to help me build datatypes
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And I usually write destructor functions to access it
(define (make-complex real imag)
  (cons real imag))

And I usually write destructor functions to access it
(define (make-complex real imag)
 (cons real imag))

(define (get-real complex)
 (car complex))

(define (get-imag complex)
 (cdr complex))
Now, define (add-complex c1 c2)
Next, define `(make-cartesian x y)`

And the associated helper functions
> (map (lambda (str) (string-ref str 0)) '("ha" "ha"))
'(#\h #\h)
(map f l) takes a function f and applies f to each element of l
[0, 1, 2]
[0, 1, 2]
Next class we will talk about…

struct

match

I/O
Intermediate Racket Programming
Tail Recursion

Tail recursion is the way you make recursion fast in functional languages

Anytime I’m going to recurse more then 10k times, I use tail recursion

(I also do it because it’s a fun mental exercise)
Tail Recursion

A function is *tail recursive* if all recursive calls are in *tail position*

A call is in tail position if it is the last thing to happen in a function
The following is not tail recursive

(define (factorial x)
  (if (equal? x 0)
      1
      (* (factorial (- x 1)) x)))

The following is tail recursive

(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x))))
The following is **not** tail recursive

```
(define (factorial x)
  (if (equal? x 0)
    1
    (* (factorial (- x 1)) x)))
```

Explain to the person next to you why this is
The following is tail recursive

(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x))))

Swap. Explain to the person next to you why this is
This isn’t merely trivia!
(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x))))

; .. Later
(factorial 2 1)
(define (factorial x acc)
    (if (equal? x 0)
        acc
        (factorial (- x 1) (* acc x)))))

; .. Later
(factorial 2 1)

>factorial 2 1
(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x))))

; .. Later
(factorial 2 1)

>factorial 2 1

>factorial 1 2
(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x)))))

; .. Later
(factorial 2 1)

>factorial 2 1

>factorial 1 2

>factorial 0 2
(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x))))

; .. Later
(factorial 2 1)

>factorial 2 1

>factorial 1 2

>factorial 0 2
(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x)))))

; ;; Later
(factorial 2 1)

> factorial 2 1

> factorial 1 2

> factorial 0 2
(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x))))

; .. Later
(factorial 2 1)

>factorial 2 1

>factorial 1 2

>factorial 0 2
(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x))))

; .. Later
(factorial 2 1)

>factorial 2 1

>factorial 1 2

>factorial 0 2

But wait!
I don’t need the stack at all!
Insight: in tail recursion, the stack is just used for copying back the results.
So just forget the stack. Just give the final result to the original caller.

Insight: in tail recursion, the stack is just used for copying back the results.
Insight: in tail recursion, the stack is just used for copying back the results. So just forget the stack. Just give the final result to the original caller.

This is called “tail call optimization”
(define (factorial x acc)
  (if (equal? x 0)
      acc
      (factorial (- x 1) (* acc x)))))

; .. Later
(factorial 2 1)
Why couldn’t I do that with this?

(define (factorial x)
  (if (equal? x 0)
    1
    (* (factorial (- x 1)) x)))

Talk it out with neighbor
Tail recursion for $\lambda$ and profit…

To make a function tail recursive…
- add an extra accumulator argument
- that tracks the result you’re building up
- then return the result
- might have to use more than one extra arg
- Call function with base case as initial accumulator

This isn’t the only way to do it, just a nice trick that usually results in clean code…
(define (factorial x)
  (if (equal? x 0)
      1
      (* (factorial (- x 1)) x)))

(define (factorial-tail x acc)
  (if (equal? x 0)
      acc
      (factorial-tail (- x 1) (* acc x))))

(define (factorial x) (factorial-tail x 1))
(define (max-of-list l)
  (cond [(eq? (length l) 1) 1]
        [(empty? l) (raise 'empty-list)]
        [else (max (first l) (max-of-list (rest l)))]))

Write this as a tail-recursive function
foldl

Like map, a higher order function operating on lists

(foldl / 1 '(1 2 3)) = (/ 3 (/ 2 (/ 1 1)))

(foldl + 0 '(1 2 3)) = (+ 3 (+ 2 (+ 1 0)))
(define (concat-strings l)
  (foldl (lambda (next_element accumulator)
       (string-append next_element accumulator))
    ""
    (reverse l)))
Challenge: use foldl to define max-of-list
**Challenge: define foldl**
Structures, Pattern Matching, and Contracts
Last time
(car
  (cdr
    (cons 3 (cons 2 '()))))
This time
5  Programmer-Defined Datatypes

New datatypes are normally created with the `struct` form, which is the topic of this chapter. The class-based object system, which we defer to Classes and Objects, offers an alternate mechanism for creating new datatypes, but even classes and objects are implemented in terms of structure types.

5.1  Simple Structure Types: `struct`

To a first approximation, the syntax of `struct` is

```
(struct struct-id (field-id ...))
```

Examples:

```
(struct posn (x y))
```

The `struct` form binds `struct-id` and a number of identifiers that are built from `struct-id` and the `field-ids`:

- `struct-id` : a `constructor` function that takes as many arguments as the number of `field-ids`, and returns an instance of the structure type.

  Example:

  ```
  > (posn 1 2)
  #<posn>
  ```

- `struct-id?` : a `predicate` function that takes a single argument and returns `#t` if it is an instance of the structure type, `#f` otherwise.

  Examples:
Use **struct** to define a new datatype
(struct empty-tree ()

(struct leaf (elem))

(struct tree (left right))
Copy these

(struct empty-tree ())

(struct leaf (elem))

(struct tree (value left right))
(empty-tree)

(leaf 23)

(tree 12 (empty-tree) (leaf 23))
Racket automatically generates helpers...

tree?

tree-left

tree-right
Write `max-of-tree`

*Use the helpers*
Pattern matching
Pattern matching allows me to tell Racket the “shape” of what I’m looking for
Manually pulling apart data structures is laborious
(define (max-of-tree t)
  (match t
    [(leaf e) e]
    [(tree v _ (empty-tree)) v]
    [(tree _ _ r) (max-of-tree r)]))
Variables are bound in the match, refer to in body

(define (max-of-tree t)
  (match t
    [(leaf e) e]
    [(tree v _ (empty-tree)) v]
    [(tree _ _ r) (max-of-tree r)]))
(define (max-of-tree t)
  (match t
    [(leaf e) e]
    [(tree v _ (empty-tree)) v]
    [(tree _ _ r) (max-of-tree r)]))

Note: match struct w/ (name params...
Define is-sorted
Can match a list of x's

(list x y z ...)

(1 2 3 4)

x = 1 y = 2 z = '(3 4)
Can match cons cells too…

(cons x y)
Variants include things like match-let
Racket has a "reader"
(read)
Racket “reads” the input one *datum* at a time
> (read)
(1 2 3)
'(1 2 3)
> (read)
1 2 3
1
> (read)
2
> (read)
3
>
Read will “buffer” its input
(read-line)
(open-input-file)
Contracts
(define (reverse-string s)
  (list->string (reverse (string->list s))))
Write out the call and return type of this for yourself
(define (factorial i)
  (cond
   [(= i 1) 1]
   [else (* (factorial (- i 1)) i)]))
What are the call / return types?
What is the pre / post condition?
(define (gt0? x) (> x 0))
(define/contract (factorial i) 
  (-> gt0? gt0?)
  (cond
    [(= i 1) 1]
    [else (* (factorial (- i 1)) i)])
Now in tail form...
(define (fac-tail i)
  (letrec ([h (lambda (i acc)
             (cond
               [(= i 0) acc]
               [else (h (- i 1) (* acc i))]]))]
    (h i 1)))
Now, let’s say I want to say it’s equal to factorial...
(define/contract (fac-tail i)
  (\->i ([x (>=/c 0)]
    [result (x) (lambda (result) (= (factorial x) result))]))
(letrec ([h (lambda (i acc)
          (cond
            [ (= i 0) acc]
            [else (h (- i 1) (* acc i))]))])
  (h i 1)))
(->i ([x (>=/c 0)])
[result (x) (lambda (result) (= (factorial x) result))])))
(define/contract (reverse-string s)
  (-> string? string?)
  (list->string (reverse (string->list s)))))
(define/contract (reverse-string s)
  (-> string? string?)
  (list->string (reverse (string->list s))))
(<=/c 2)
打败/C takes an argument \( x \), returns a function \( f \) that takes an argument \( y \), and \( f(y) = \#t \) if \( x \leq y \).
\(<=/c\) takes an argument \(X\), returns a function \(f\) that takes an argument \(y\), and \(f(y) = \#t\) if \(x \leq y\).

(Note: \(<=/c\) is also doing some bookkeeping, but we won’t worry about that now.)
Challenge: write \leq/
Three stories
(define/contract (call-and-concat f s1 s2)
  (-> (-> string? string?) string? string? string?)
  (string-append (f s1) (f s2))))

(define (reverse-string s)
  (list->string (reverse (string->list s))))
Scenario: you call call-and-concat with reverse
Scenario: you call call-and-concat with reverse, 12, and "12"
Now define

(define/contract (call-and-concat f s1 s2)
  (-> (-> string? string?) string? string? string?)
  (length (string-append (f s1) (f s2)))))
Now define

(define/contract (call-and-concat f s1 s2)
  (-> (-> string? string?) string? string? string?)
  (length (string-append (f s1) (f s2))))

What went wrong?
Now define

\[
\text{(define/contract (call-and-concat f s1 s2) }
\text{(-> (-> string? string?) string? string? string?) }
\text{(length (string-append (f s1) (f s2))))}
\]

What went wrong?

Who is to blame?