Sets and Tries

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Set (ADT)

Many possible implementations of sets!

- `add(set,element)`
- `contains(set,element)`
- `union(set,set)`
- `intersection(set,set)`

- Hash tables
- Balanced BSTs
- Lists

Sets as Hash Tables

- Exercise: How would you perform:
  - Insert
  - Lookup
  - Union
  - Intersection
- What is runtime (big-O) of each, assuming good hash function, m buckets, and *at most* n values in each set

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  - Lookup
  - Union
  - Intersection

Assume you wanted a persistent version of each operation, how do runtimes change?

- What is runtime (big-O) of each, assuming good hash function, m buckets, and *at most* n values in each set

Upshot: hash tables decent at many common set operations

(But better implementations exist)

Sets as BSTs

(Imperative version..)

- Exercise: How would you perform:
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  - Lookup
  - Union
  - Intersection
- What is runtime (big-O) of each? Assume n elements in set

Sets as BSTs

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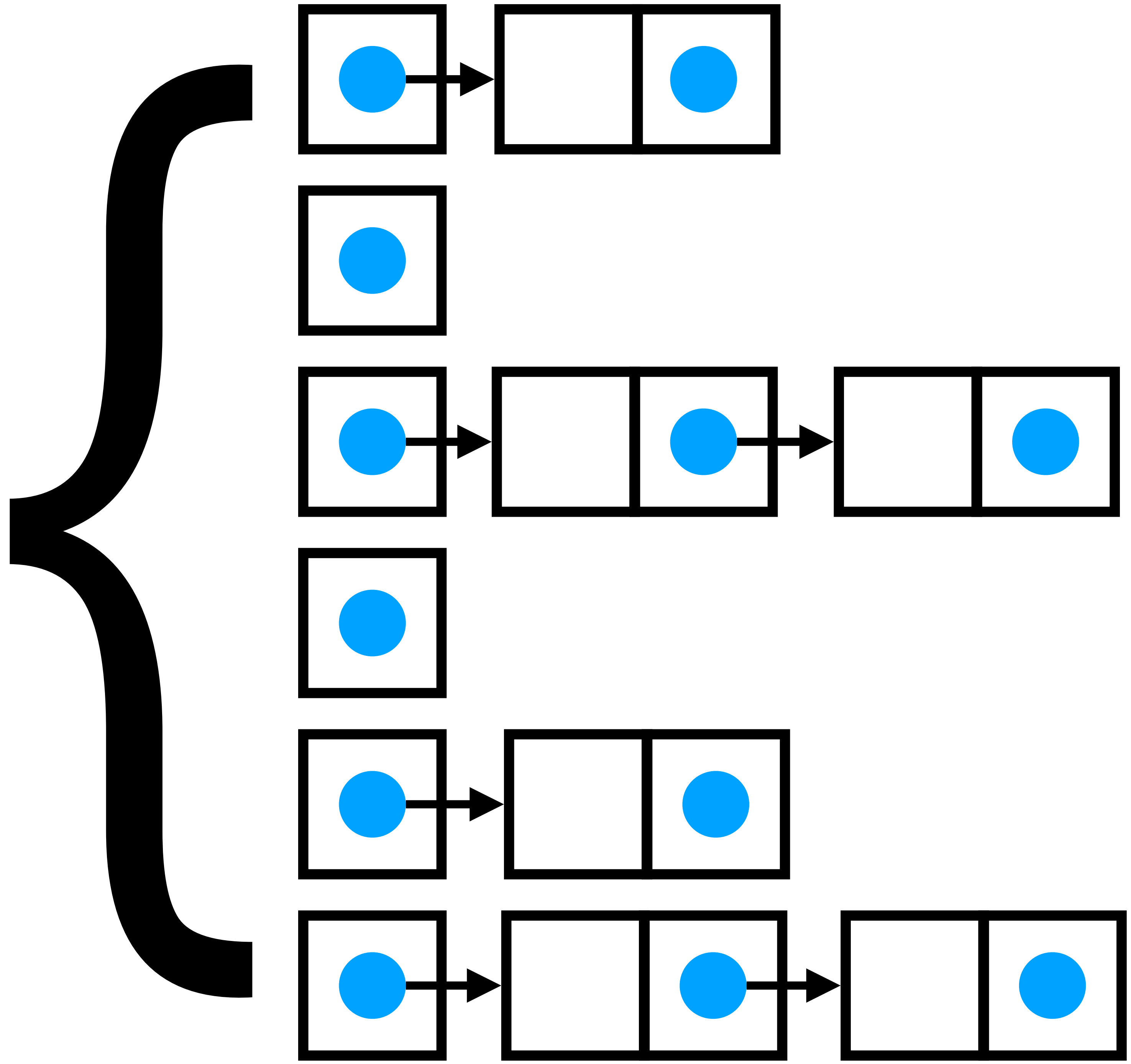
Assume you wanted a persistent version of each operation, how do runtimes change?

Upshot: BSTs give us better persistent hash behavior

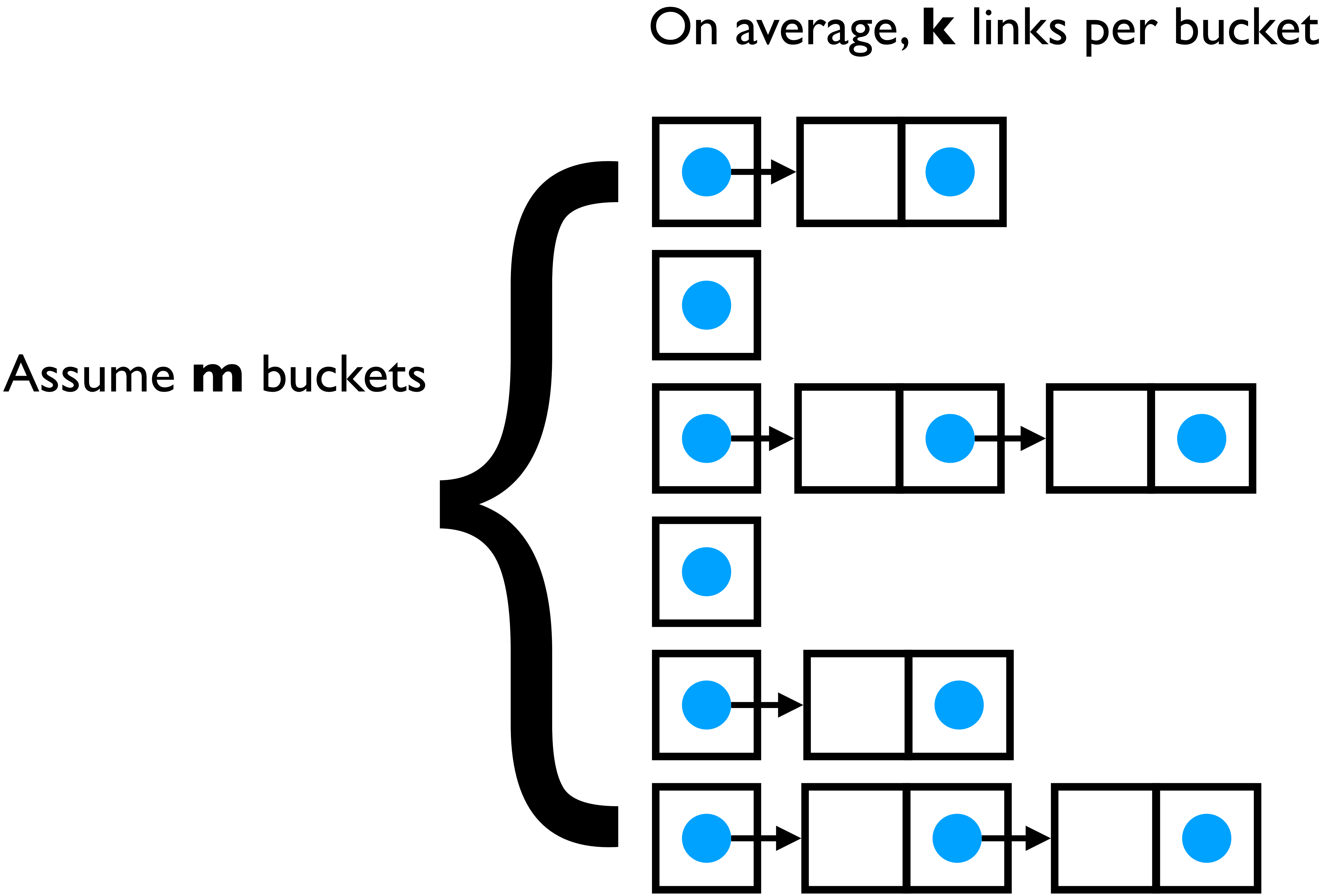
(But **still** better implementations exist!)

On average, **k** links per bucket

Assume **m** buckets



Observation: if you have m*k items in table, lookup takes ~k time



Question

If you want to lower per-item lookup time, what do you do?

Observation

More buckets = faster lookup

(To a point… Then you bottom out)

Exercise

- Load ~500k words into dictionary
- Then, perform ~500k lookups
  - Not good benchmark of real-world use b/c uniform dist
- Expectation: bigger hash table = lower lookup time
- **What real-world problem does this solve..?**

```
# Load `words.txt` into a hash table of size s
def loadIntoTable(s):
    print("loading words into table...")
    hashTable = HashTable(s)
    with open("words.txt", "r") as ins:
        for line in ins:
            words.append(line)
            hashTable.insert(line,True)
    return hashTable

# Look up each word in the hash table
def lookupWords(table):
    print("looking up all words in dictionary")
    for word in words:
        table.lookup(word)
```

Vary s by 10k, 20k, ...

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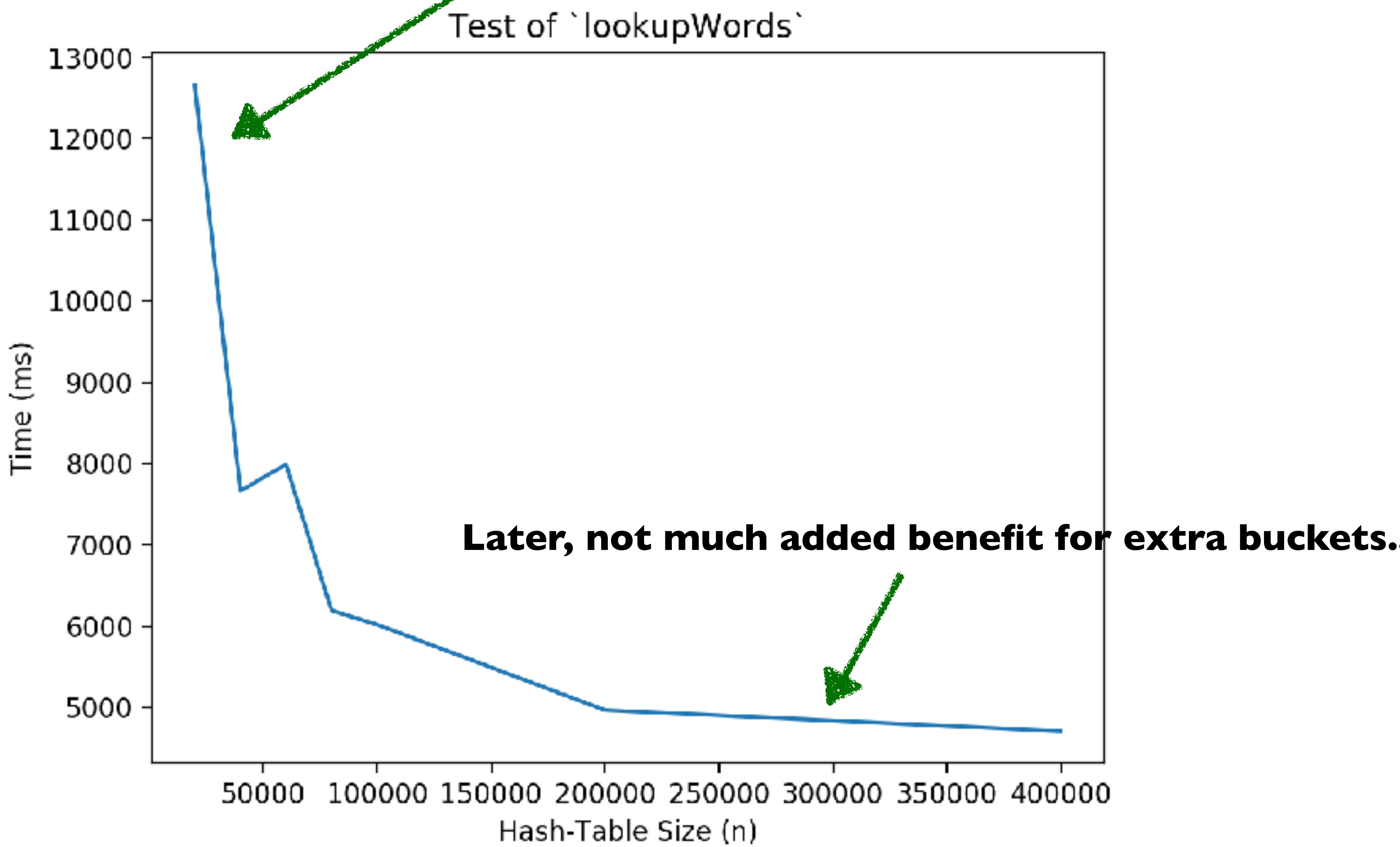
At first, not enough buckets to compensate



Question

Why not just always use 100k buckets?

Observation: An optimal hash table requires knowing a priori the number of elements stored in it

If we're using less of the hash table than we need, we're wasting a lot of memory just on the buckets

Enter the trie…

A trie is a **suffix tree** that *compactly* represents sets of strings

Let’s say we want to represent the following set…

{“Ali”, “Aly”, “Al”}

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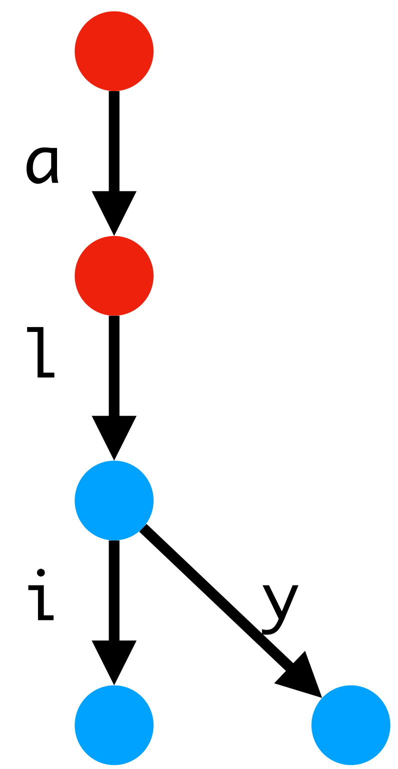
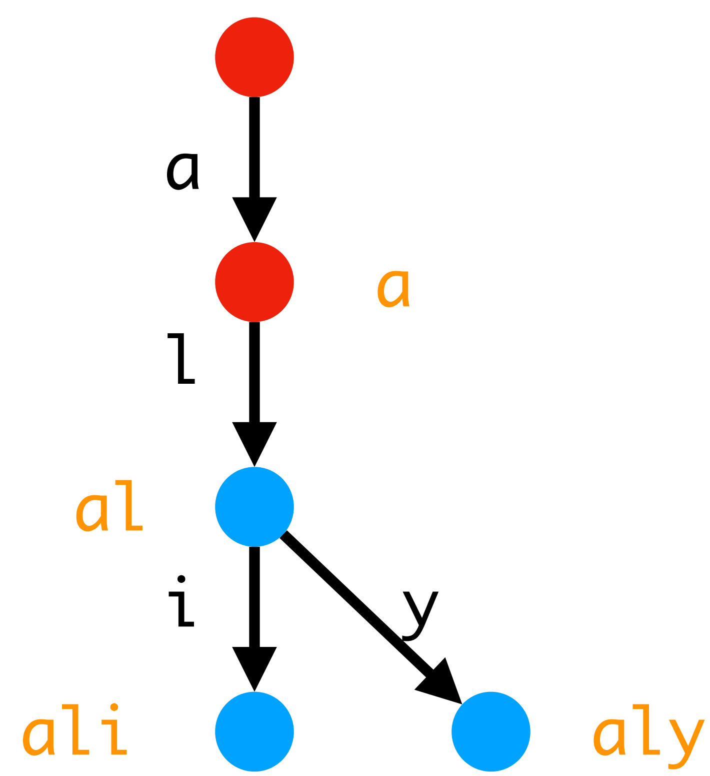
{“ali”, “aly”, “al”}



Trie shares **common prefixes**. In this example, blue nodes indicate that the element is in the set

Get value by traversing down spine…



{"ali", "aly", "al"}

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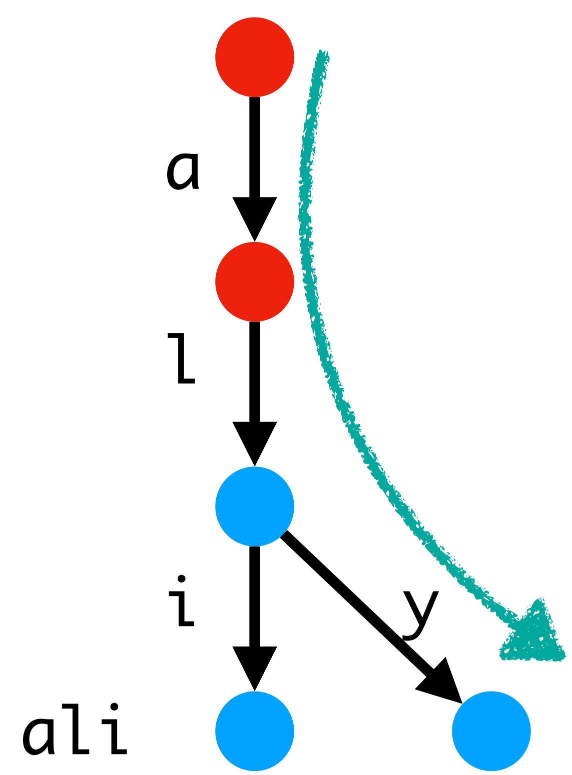


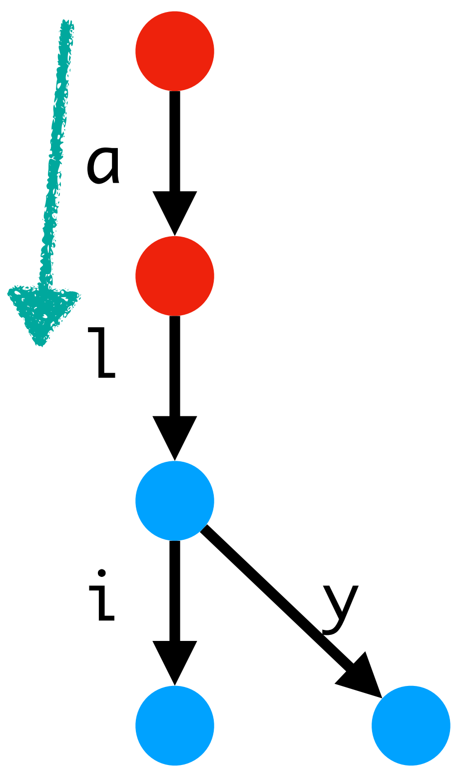
Start at root, read…
a, l, y

{"ali", "aly", "al"}

Trie shares **common prefixes**. In this example, blue nodes indicate that the element is in the set

Get value by traversing down spine…

Red node = no data at that node



{"ali", "aly", "al"}

Draw Example Trie For…

- {“b”, “ba”, “bac”}
- {“alice”, “alicia”, “alejandro”}
- {“alice”, “bob”}

Building Tries

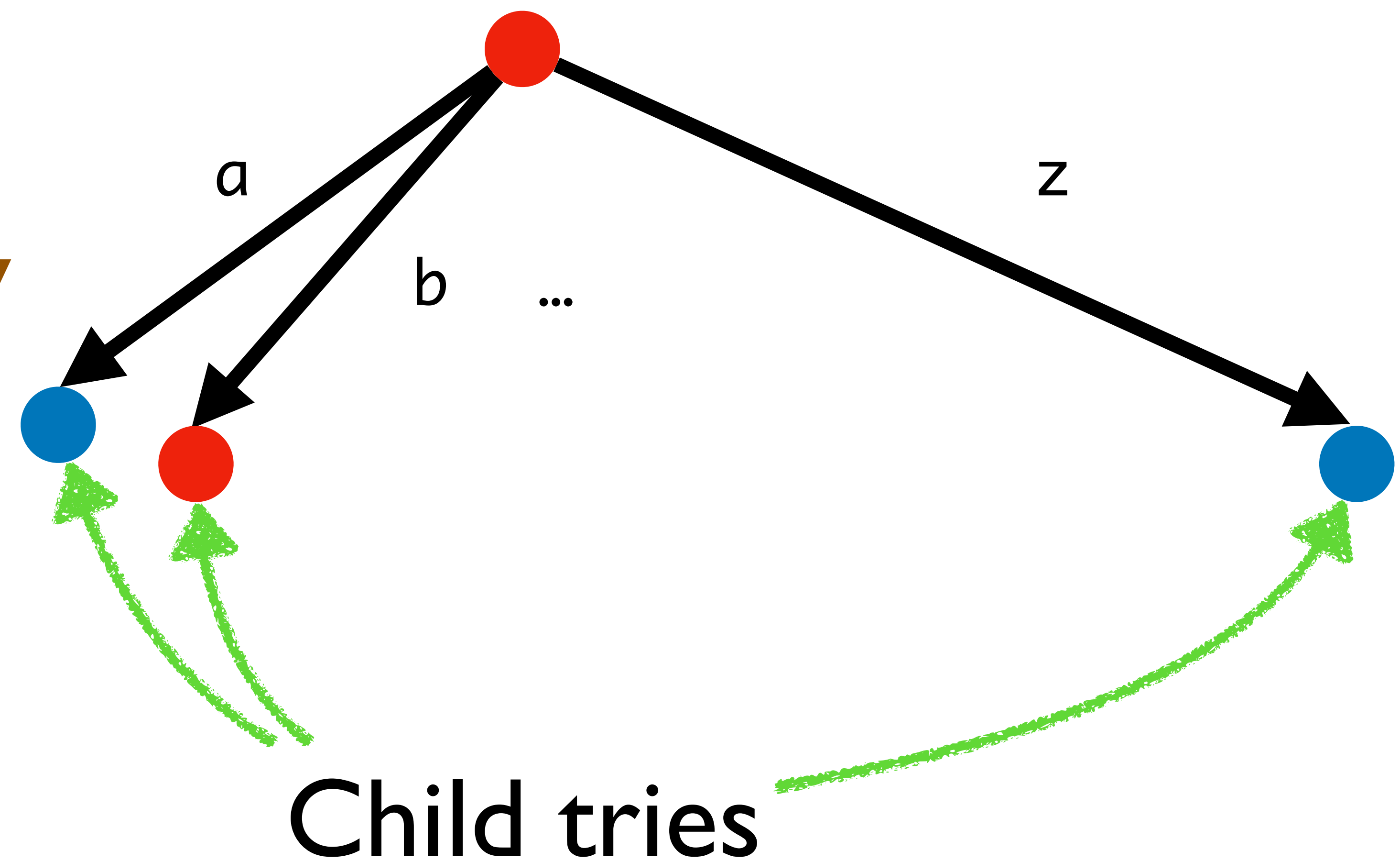
- Let's say we want to build tries for strings in the English lowercase alphabet only
  - I.e., 26 characters
  - Obvious problems with this we will fix later (e.g., "José")

Building Tries

- Insight: represent trie as Node with 26 children buckets

Represent as array of size 26

Lookup-next constant time via random access

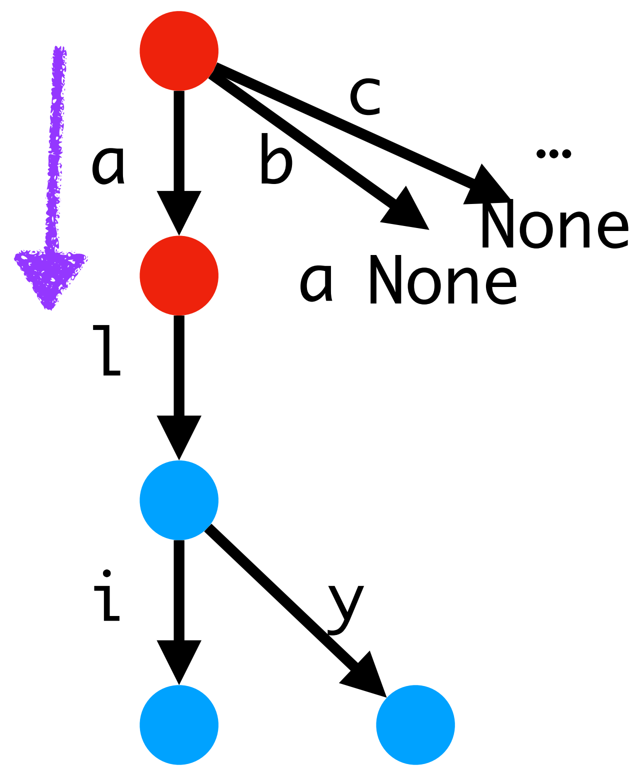


```python
class Trie:
    def __init__(self, buckets):
        self.content = False
        self.contents = [None] * buckets
        self.buckets = buckets

    def bucket(self,chr):
        return ord(chr) - ord('a')
```

As is common in data structures, I've just shown one example formulation here, other equivalent ones exist..

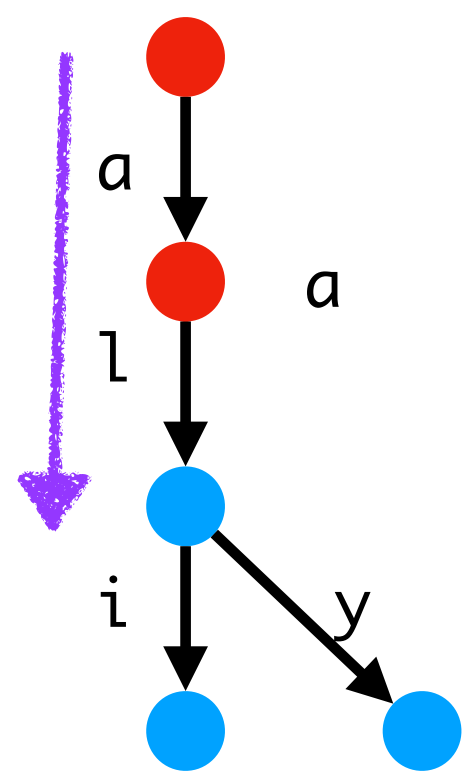
Trie Lookup

Let's lookup aly



- Start at root
- Go to "a" bucket

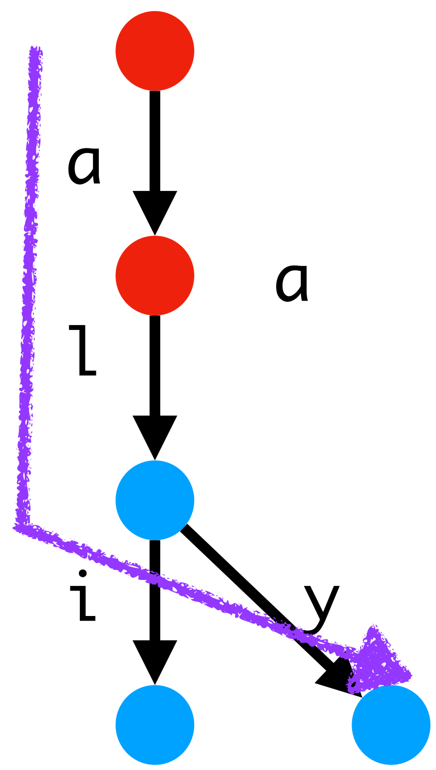
Trie Lookup



Let's lookup aly

- Start at root
- Go to "a" bucket
- Keep going to l

Trie Lookup

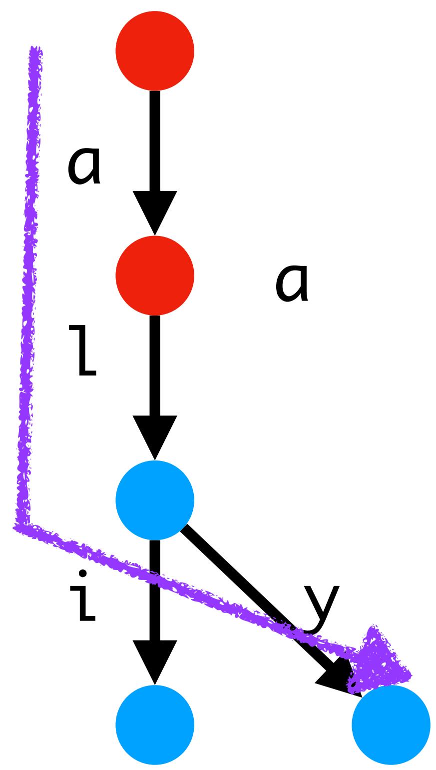


Let's lookup aly

- Start at root
- Go to "a" bucket
- Keep going to l
- Keep going to y

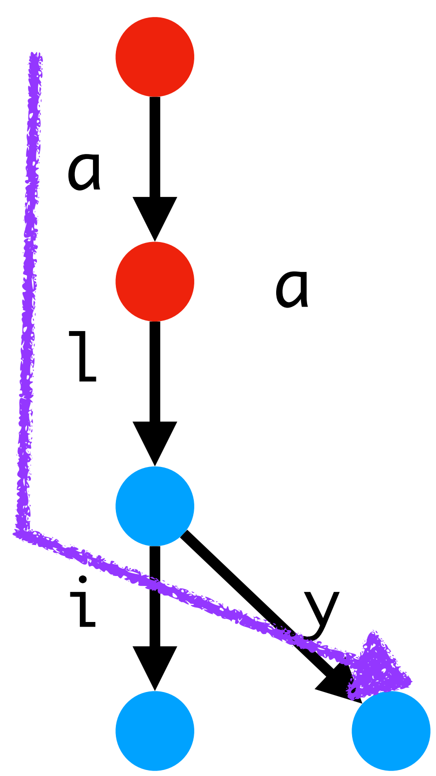
Trie Lookup

Let's lookup aly



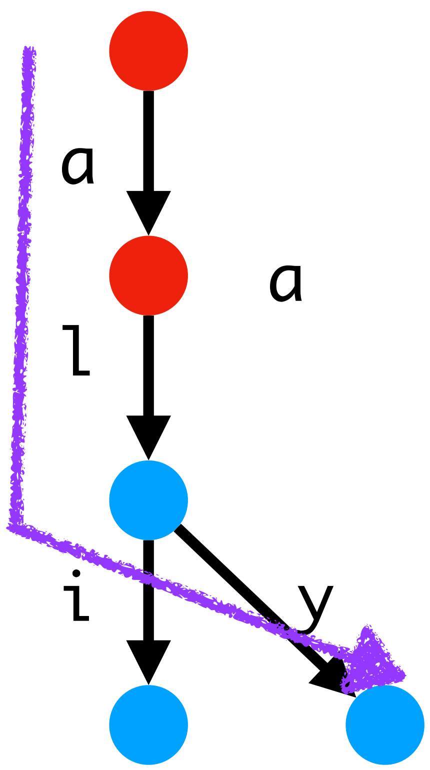
- Start at root
- Go to "a" bucket
- Keep going to l
- Keep going to y
- Return color == blue

Trie Lookup



What is running time of `lookup`?

Trie Lookup



What is running time of `lookup`?

O(len(key))

Exercise: Write pseudo-code for **lookup**

```
def lookupHelper(self,string,i,m):
    if (i >= m):
        return self.content
    else:
        bucket = self.contents[self.bucket(string[i])]
        return bucket != None
               and bucket.lookupHelper(string,i+1,m)

def lookup(self,string):
    return self.lookupHelper(string,0,len(string))
```

Can write slightly-more-optimized version of this with loops…

Exercise: Write pseudo-code for **insert**

Construct new child trie if one doesn't exist!

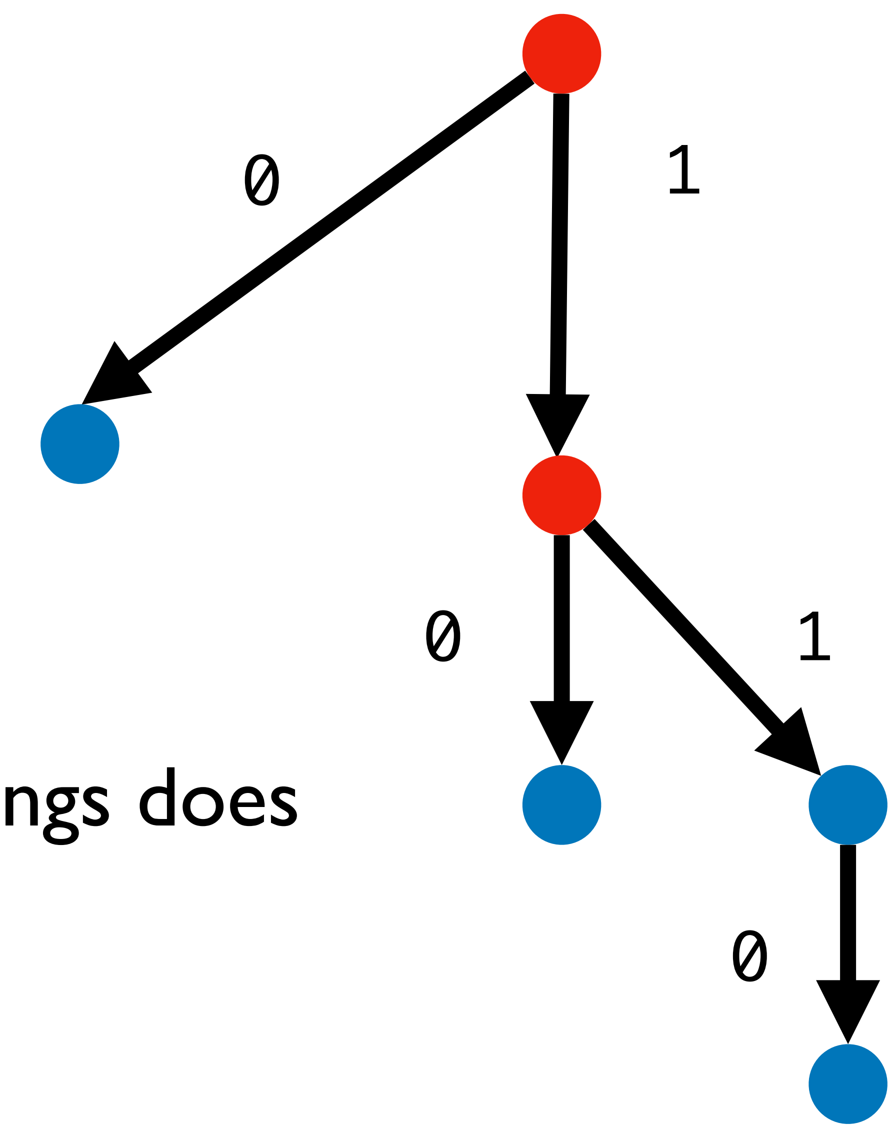
```
def insertHelper(self,string,i,m):
    if (i >= m):
        # Set this bucket to True
        self.content = True
    else:
        if (self.contents[self.bucket(string[i])] == None):
            self.contents[self.bucket(string[i])] = Trie(self.buckets)

        self.contents[self.bucket(string[i])].insertHelper(string,i+1,m)

def insert(self,string):
    self.insertHelper(string,0,len(string))
```

Question

What would persistent insert look like for tries..?

Binary Tries

- **Insight:** one simple lexicographic order is binary numbers



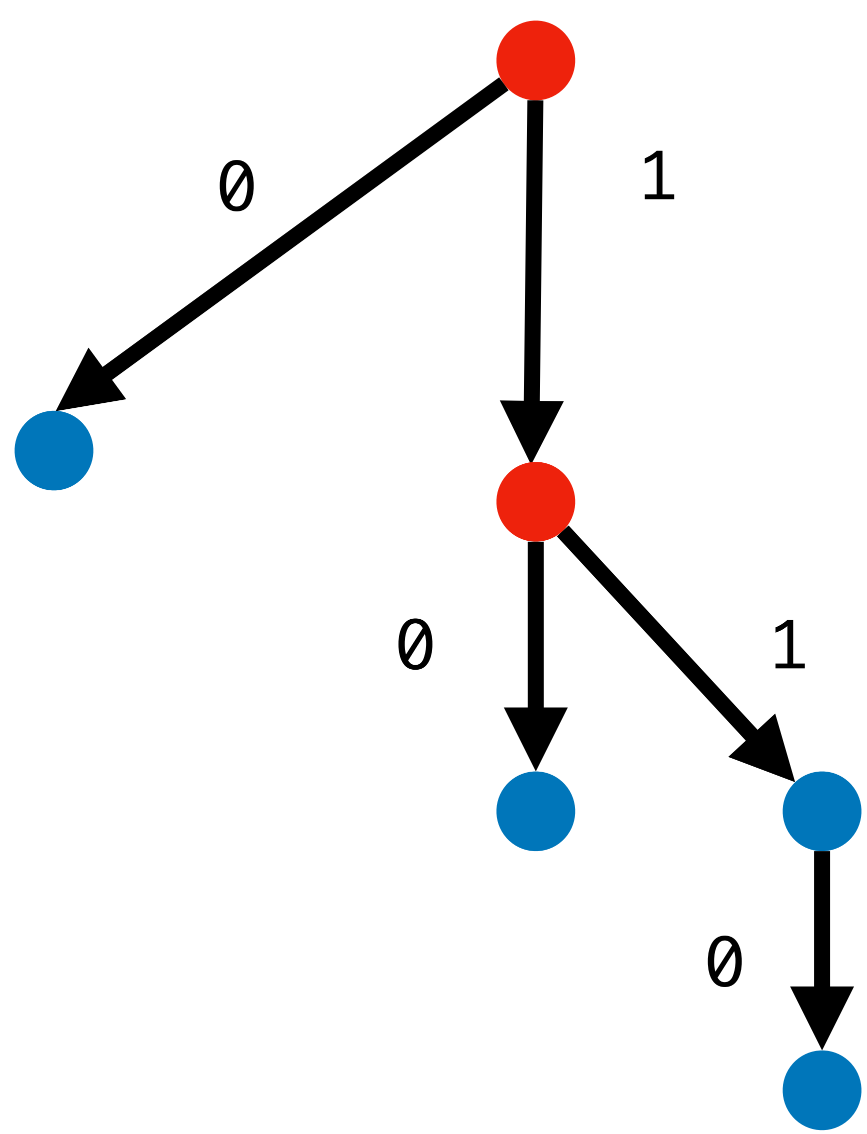
What set of binary strings does this trie represent?

Binary Tries

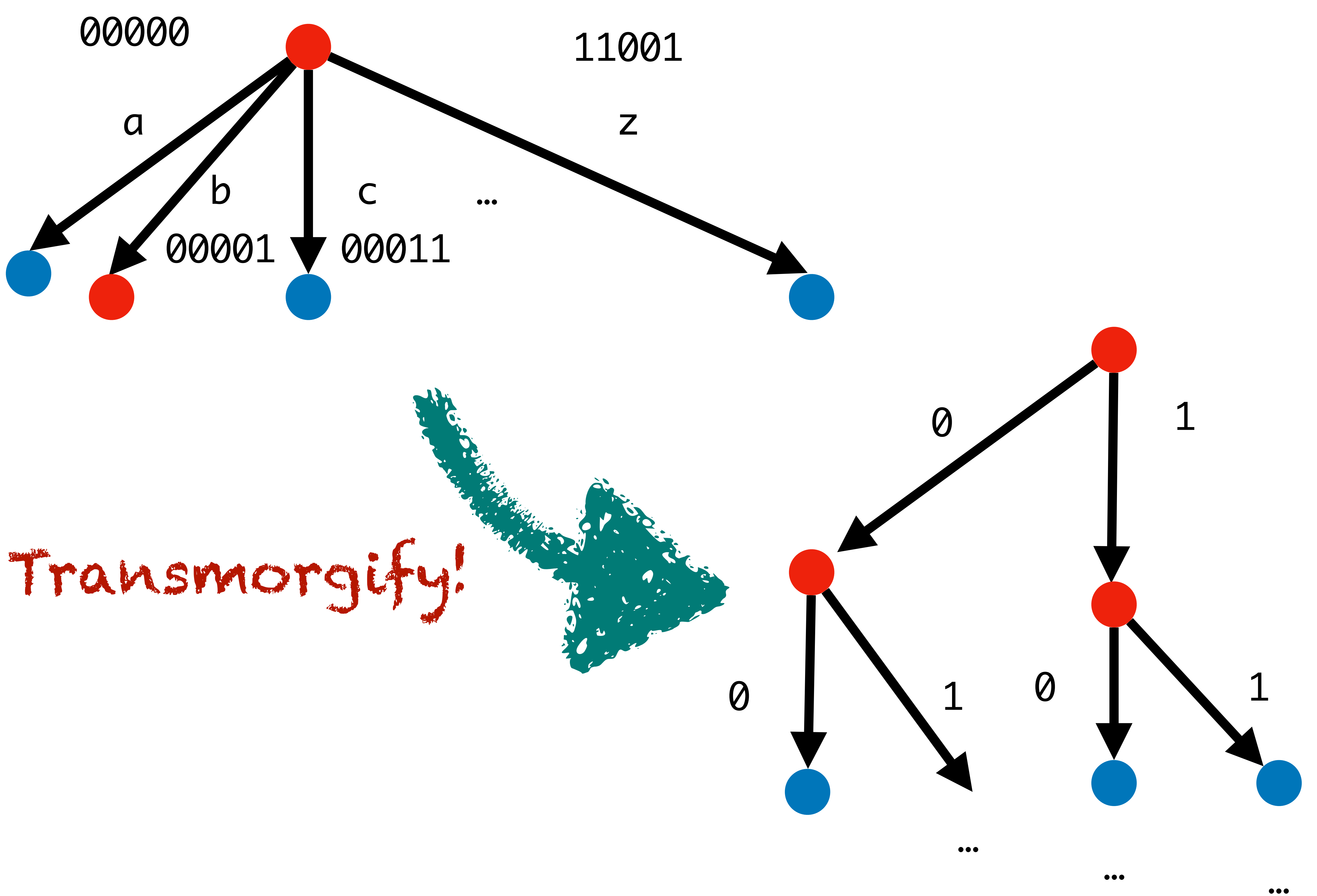
Question: in general, are binary tries a good idea?

Assuming random binary strings: better / worse than binary tree?

Assuming binary strings w/ common prefix?



Because we can treat **any** alphabet as the binary alphabet with the necessary transformations, binary tries are **always** an option!



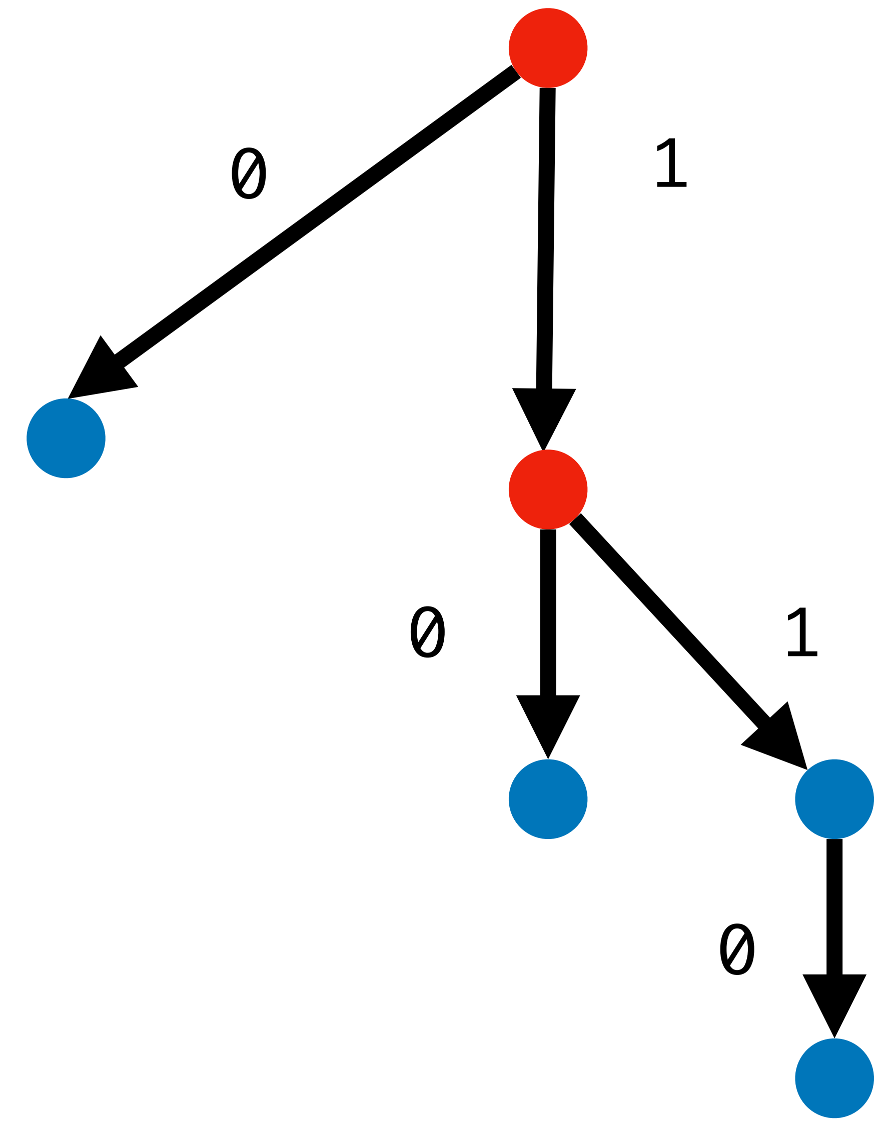
Kris speaks extemporaneously about the cache…

Binary Tries: Crummy Cache?

Your computer "caches" recently-used memory

Every time your computer needs to touch memory it hasn't seen recently it's slower

Upshot: binary trie causes potentially-lots of memory access



Tries vs. BSTs

- Trie leverages prefix-ordering—e.g., lexicographic
  - E.g., for dictionaries, O(len(word)) rather than O(log(words))
  - Also, in practice, *most words are small*, so this is even better!
- Downside: not always useful prefix ordering
  - E.g., storing floating-point numbers in a trie is naïve
- Tries **do** use more memory when implemented naively!

Bait + Switch

- First motivated tries by saying they saved memory—but tries still allocate lots of potentially-empty buckets
- What gives!?
  - Are tries more "compact" (memory efficient) vs hash tables?
  - Under what circumstances?

Going into the Future

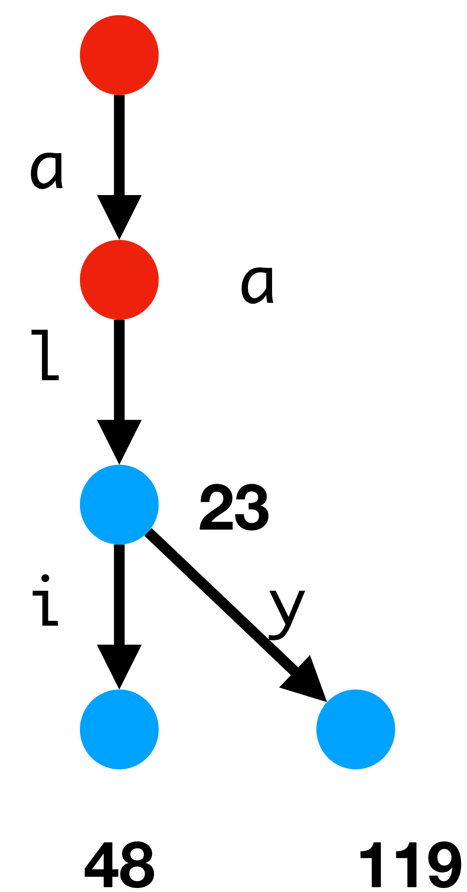
- Tries: useful representing sets w/ dense common-prefixes
- Tries are good **adaptive** data structures: "resize" automatically
  - (Hash Tbls don't do this! "Fill up" with data, becomes slow)
- As we implemented them now, not amazing performance:
  - But, **basis** for many other *optimized* implementations

- 10000 race participants are assigned numbers from 1 to 10000, but some drop out the day of the race. I would use a (**trie** / hash table)

- 2000 500-character strings come in over the network and need to be remembered, there is no apparent structure to their contents (trie / **hash table**)

Sets vs. Maps

- Most of these data structures can be used to store sets or maps (i.e., key/value associations)
- Trivial impl of set from map: just make key `True`
- To implement map from set (often): add place for value

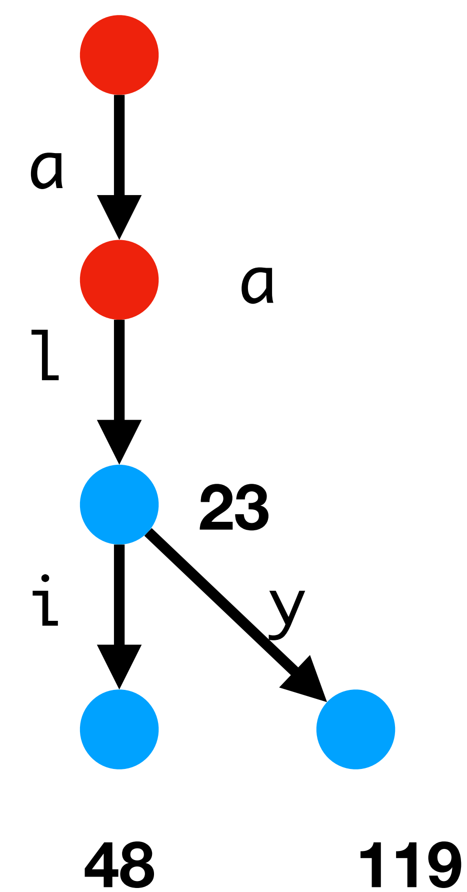


{al |-> 23, ali |-> 48, aly |-> 119}

Sets vs. Maps

Can often get "cheaper" implementation by playing low-level tricks if we know it's just a set versus a map

(E.g., bitmaps for efficiency)



Deciding Between Them

- **BST**: elements not prefix-oriented, want adaptive memory usage, okay with imperative data structure
  - Self-balancing trees (necessary for O(log(elems)) behavior) like R/B and AVL are imperative
- **Trie**: prefix-oriented data, want persistence, ok w/ more overhead than BST
- **Hash table**: You know roughly how much data you have (to avoid resizing), keys unordered, don't need persistence

Still not fast as it could be!

Come back next time for **bloom filters**

(Blazing-fast probabilistic set)

Exercises (for exam)

- Think about how you would implement union / intersection
    - For **hash tables**
    - For **BSTs**
    - For **Tries**
- Which one is best in principle? Which do you suspect would be best in practice?