Order of evaluation, and the stack

CIS 352 — Spring 2020

The stack

- Some expressions can be evaluated in O(1) time, regardless of the values involved; a subset are **atomic expressions**.
    - (cons x (cdr lst)) is an O(1) operation, but is made of *several* atomic steps: evaluate x, evaluate lst, evaluate (cdr lst), call cons to build the new cons-cell, return…
    - Variable references (e.g., to x, lst, cons) are considered **atomic expressions** that take just one conceptual step. A lambda expression is also considered atomic.
- Other, complex, expressions, such as function invocation may generally take unbounded time.
    - The stack stores pending data while this work occurs.

The stack

- Consider `(f (g x) (h y))`—how is this evaluated?
  - f is evaluated atomically
  - (g x) is evaluated as the argument to this func. value
    - g is evaluated atomically; then x
    - The value of g is applied on the value of x; …; returns
  - (h y) is evaluated as a second argument
    - h is evaluated atomically; then, y
    - The value of h is applied on the value of y; …; returns
  - The value of f is applied on g's and h's return values

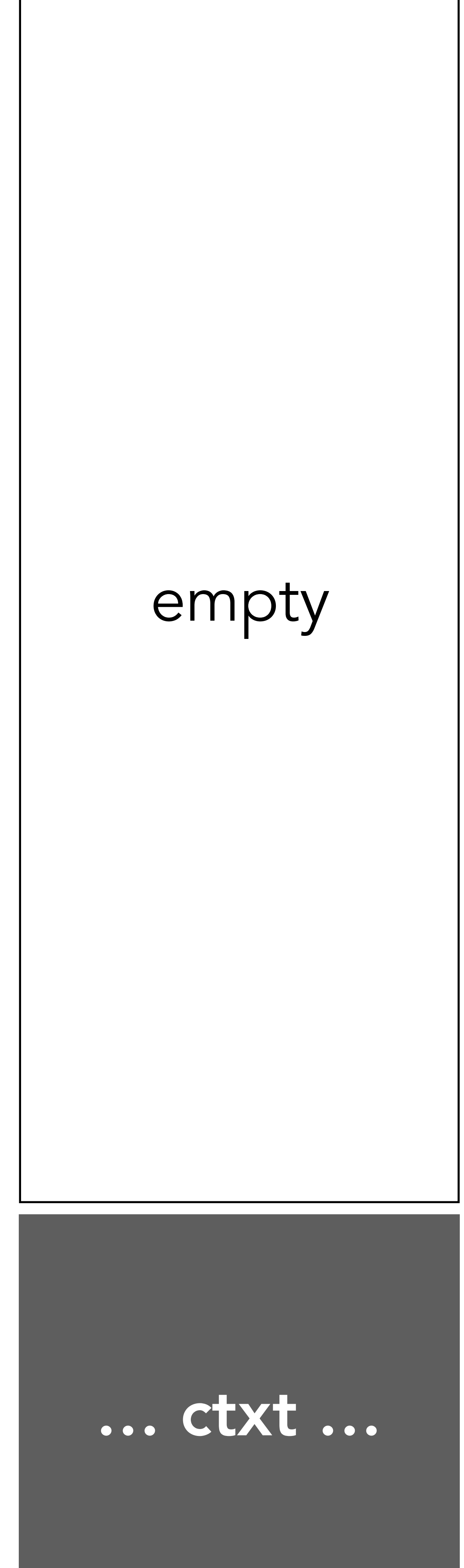
The stack

- Consider `(f (g x) (h y))`—how is this evaluated?
  - While the call (g x) is being evaluated, we need to remember a few things: the value of f just evaluated, the expression (h y) to be evaluated next; while the call (h y) is evaluated, we need to save the value of f and (g x).
    - These values are saved on the stack!
      - As calls&returns form a proper nesting structure, we want to store such pending values in LIFO order.
  - Implemented well, using a stack lends itself to improved cache performance as values used together, sit together.

The stack

`(f (g x) (h y))`

The expression is reached from a caller or surrounding expression (called the **eval. context** / call ctxt.)

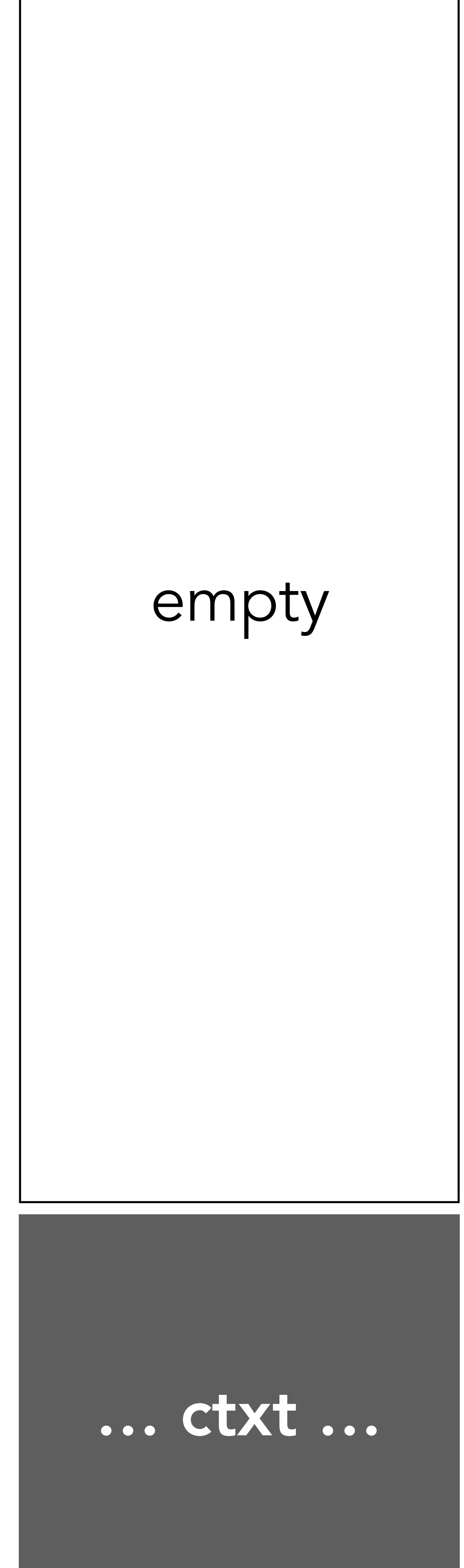


The stack

```
(f (g x) (h y))
```

Control (the current expression) steps to evaluate the subexpression in **call position** / function position.

The value of f can be evaluated atomically.

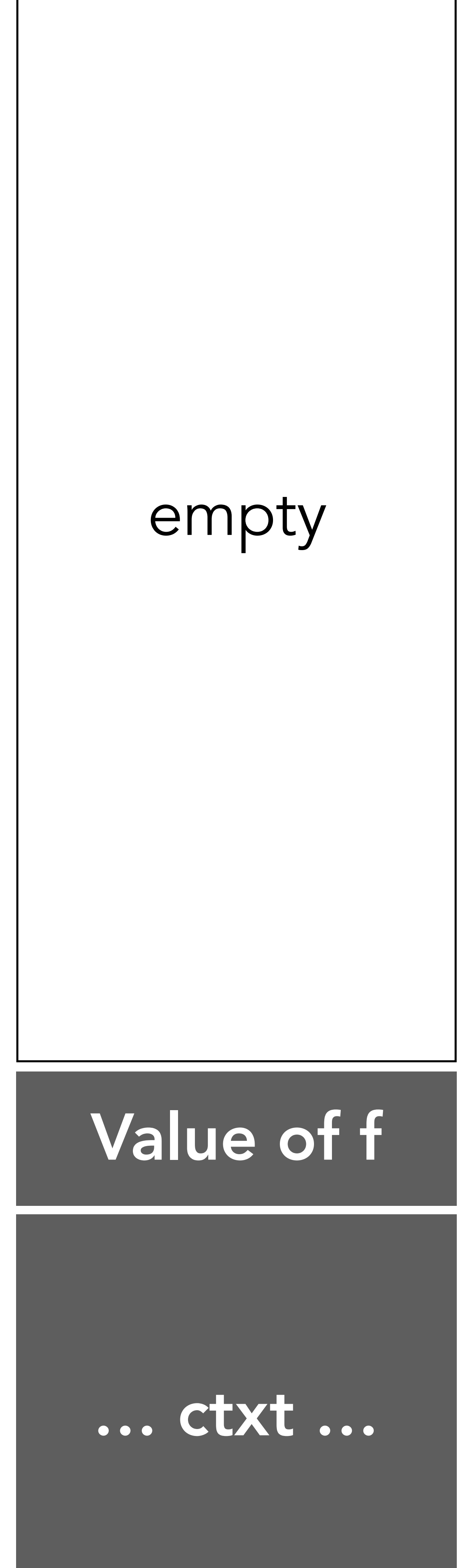


The stack

```
(f (g x) (h y))
```

Control (the current expression) steps to evaluate the first subexpression in argument position, (g x). The value for f is saved on the stack.

Its arguments must be evaluated first.

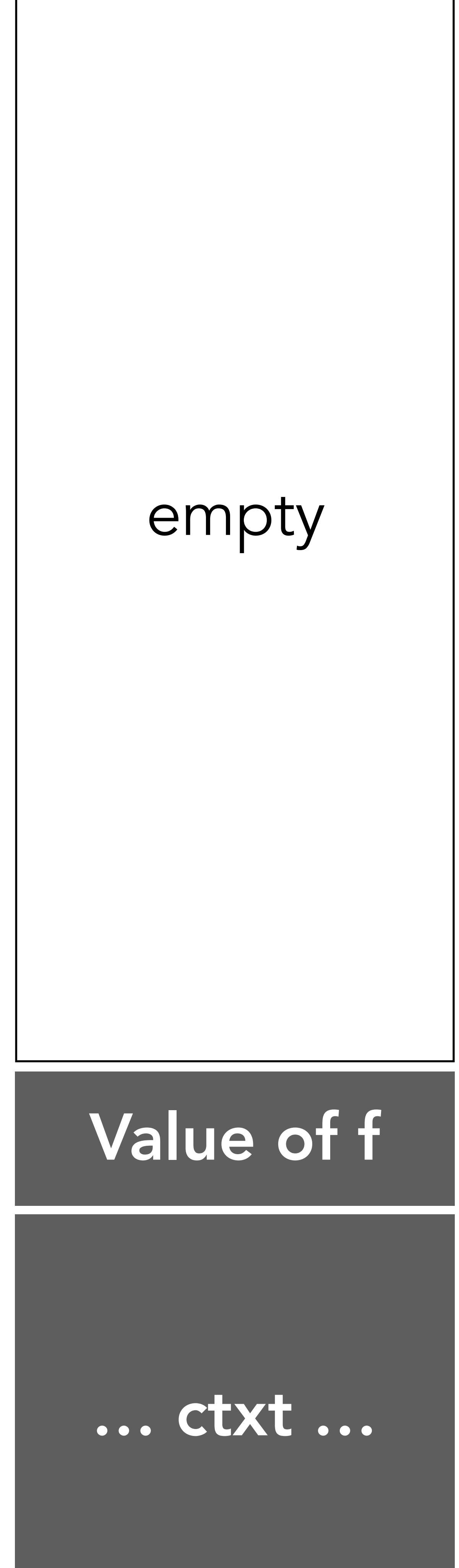


The stack

```
(f (g x) (h y))
```

Control (the current expression) steps to evaluate the subexpression in call position.

The value of g can be evaluated atomically.

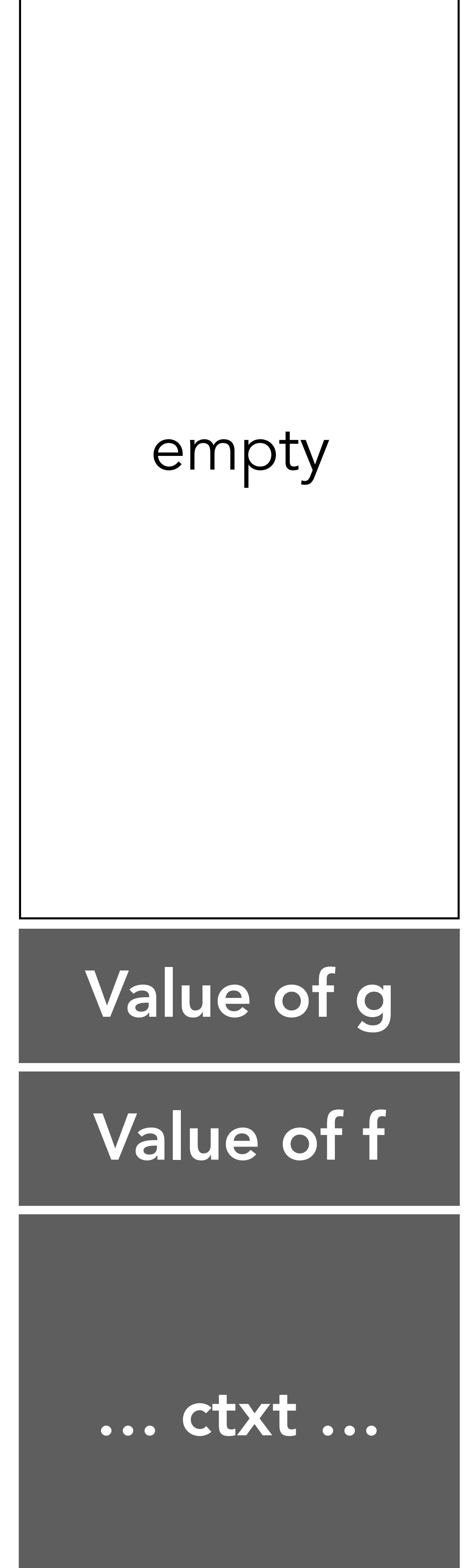


The stack

```
(f (g x) (h y))
```

Control steps to evaluate the subexpression in argument position. The value for g is saved on the stack.

The value of x can be evaluated atomically.

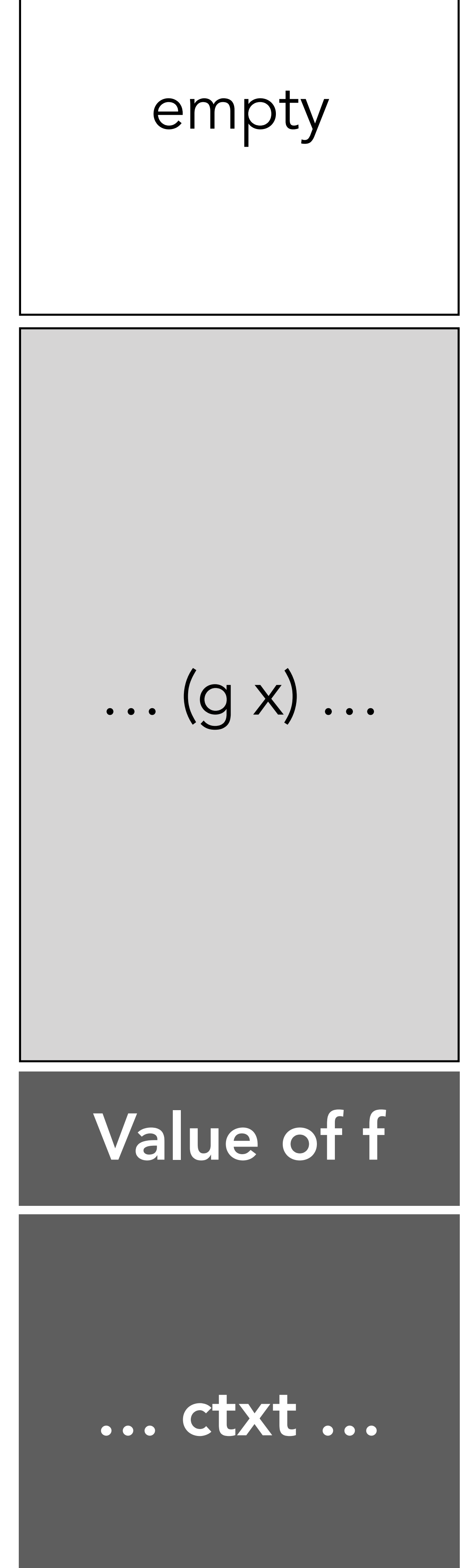


The stack

```
(f (g x) (h y))
```

The value of x just evaluated, and the value of g saved just before, can now be used to apply g on x, leading to an unbounded amount of work.

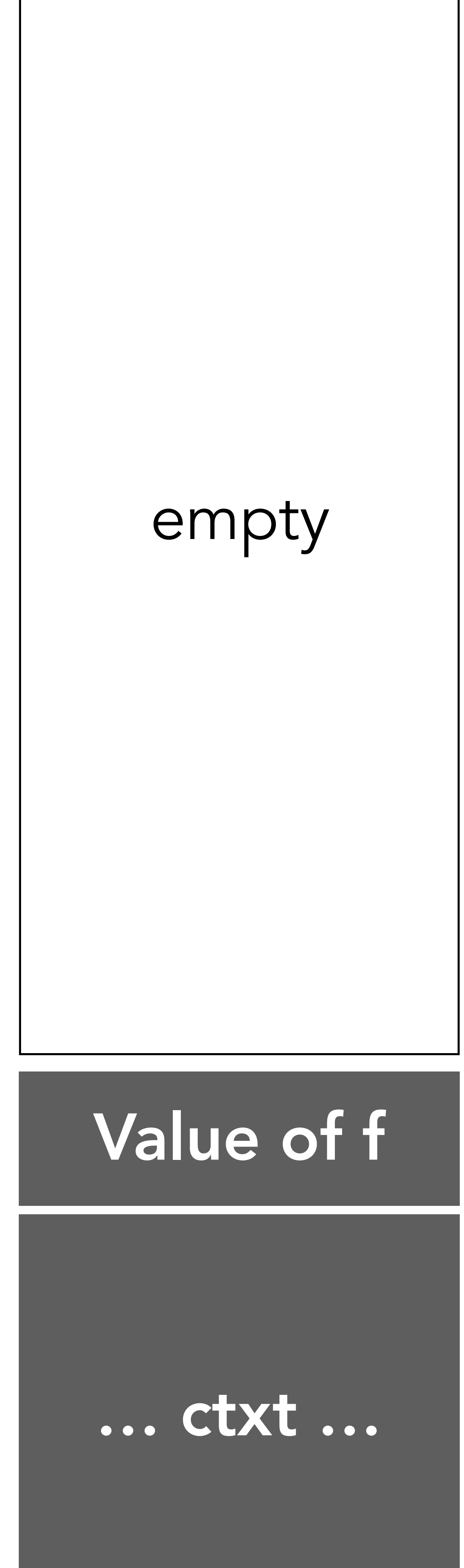
This may involve any number of pending expressions being saved and eliminated atop the stack.



The stack

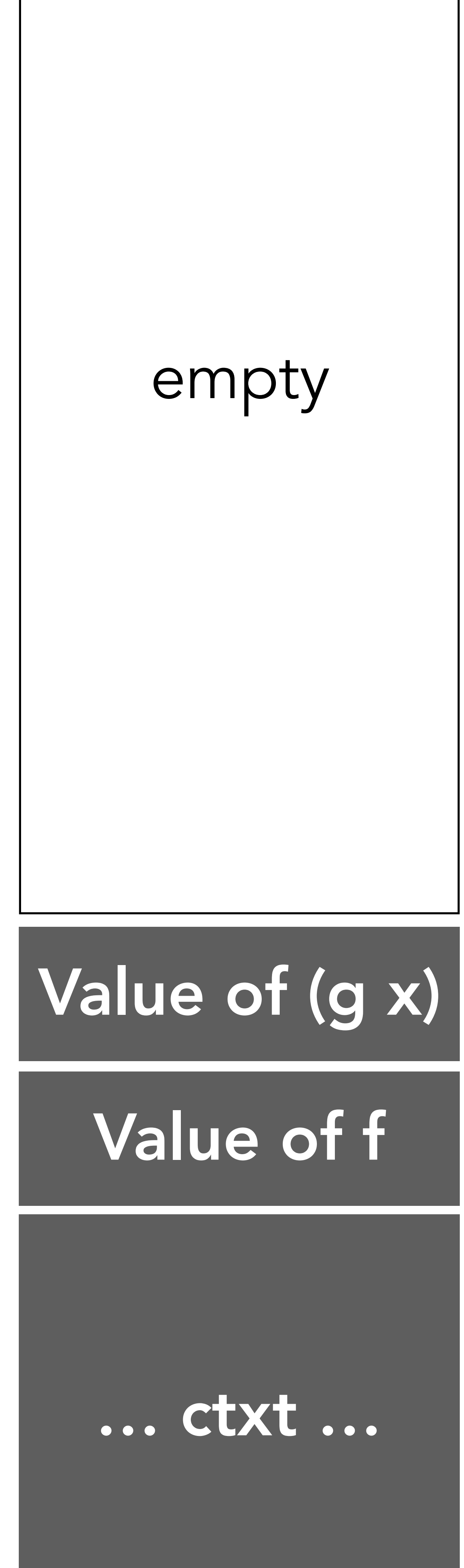
```
(f (g x) (h y))
```

g returns with a value that must
be saved on the stack while the final
argument expression of (f …) is evaluated.



The stack

```
(f (g x) (h y))
```

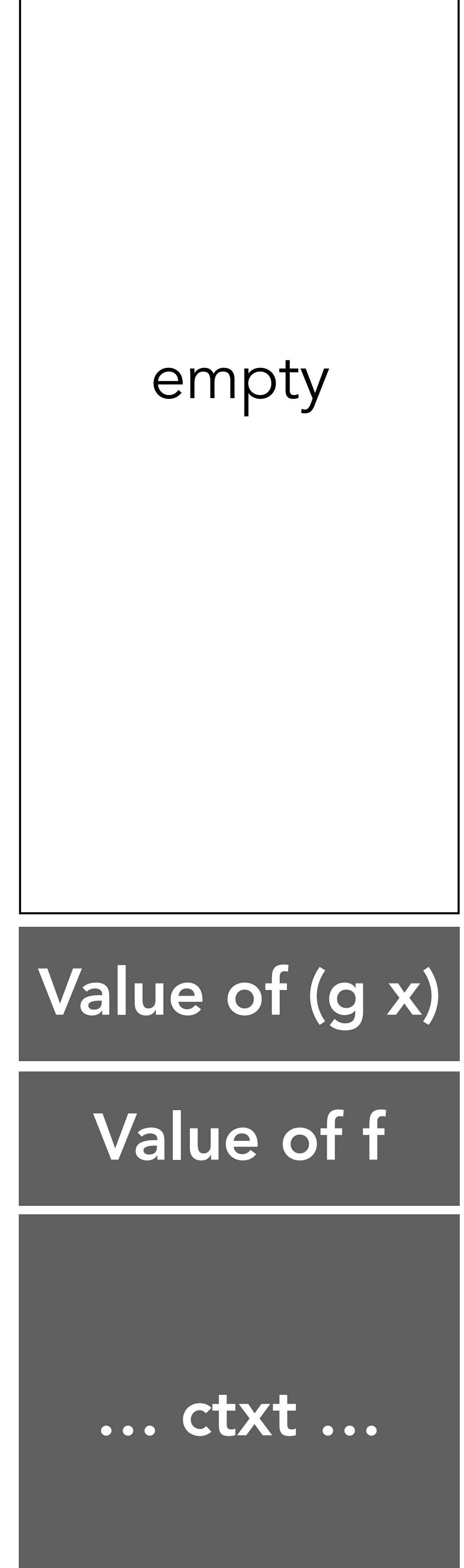


Control (the current expression) steps to evaluate the second subexpression in argument position, (h y). The value for (g x) is now saved on the stack.

The stack

```
(f (g x) (h y))
```

Control (the current expression) steps to evaluate the subexpression in call position.

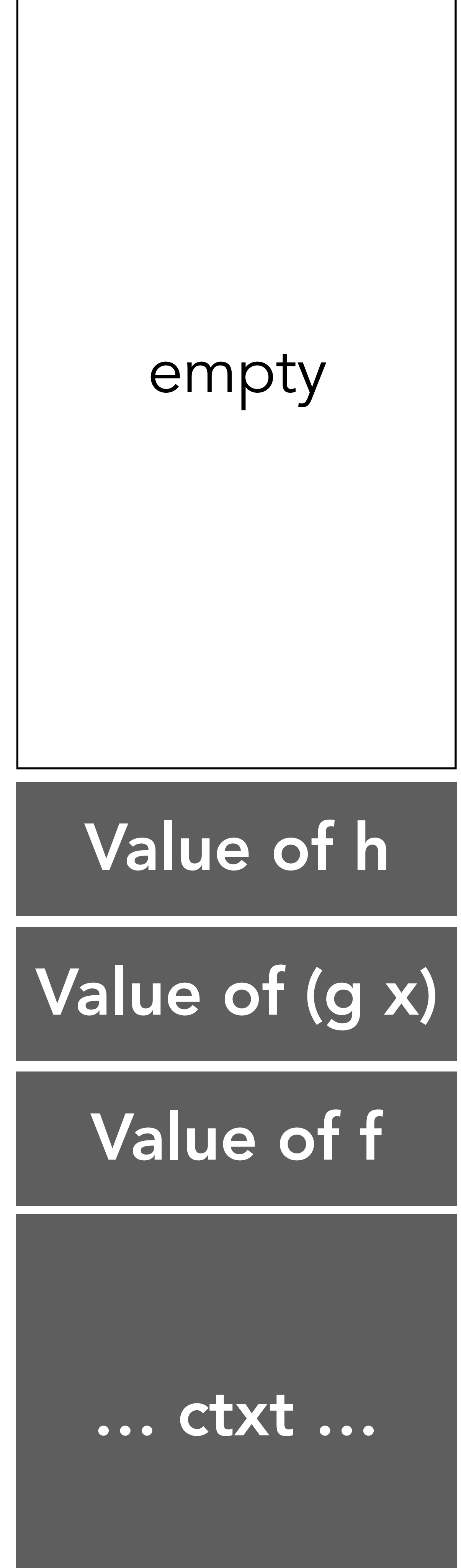


The stack

```
(f (g x) (h y))
```

Control steps to evaluate the subexpression in argument position. The value for h is saved on the stack.

The value of y can be evaluated atomically.

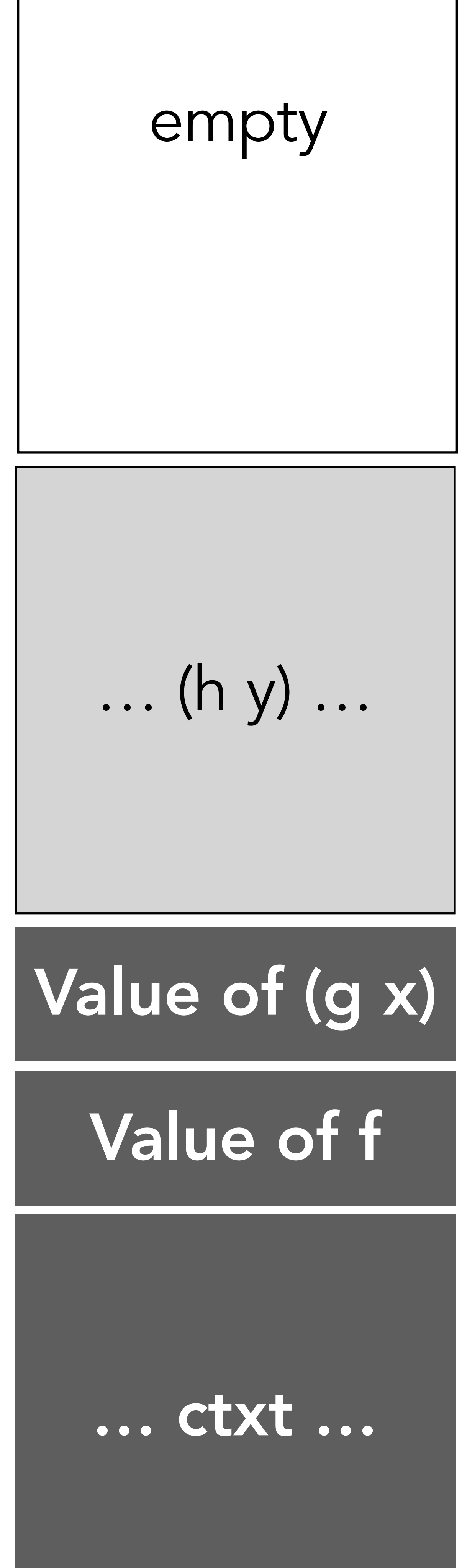


The stack

`(f (g x) (h y))`

The value of y just evaluated, and the value of h saved just before, can now be used to apply h on y, leading to an unbounded amount of work.

This may involve any number of pending expressions being saved and eliminated atop the stack.

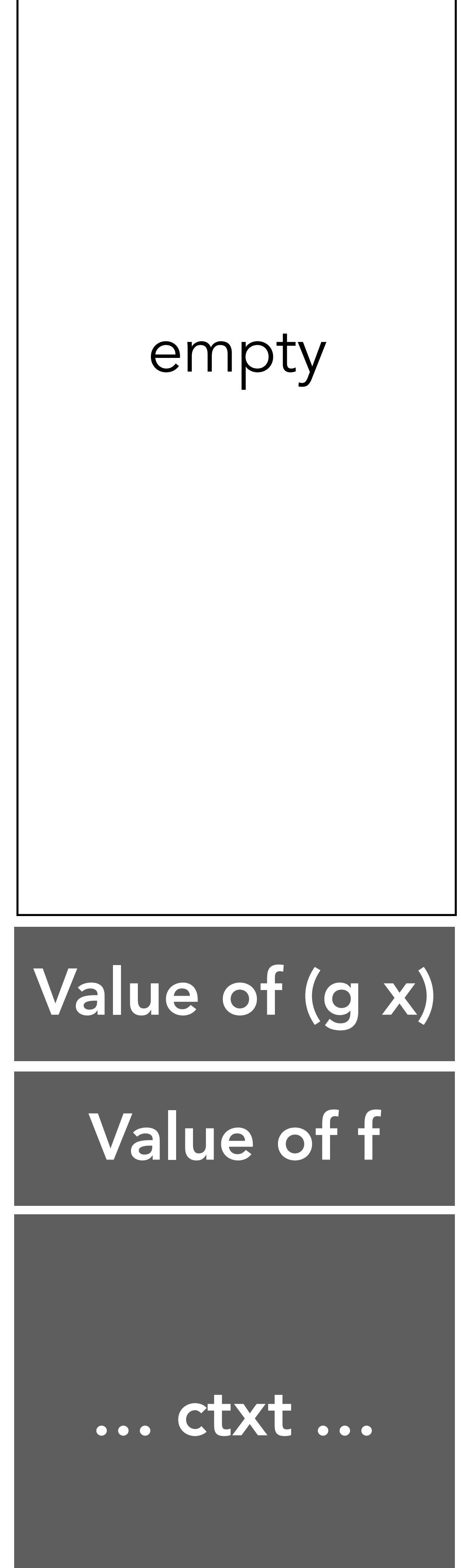


The stack

```
(f (g x) (h y))
```

h returns with the final argument value for the original call to f

The value of f, saved on the stack, is applied on the value of (g x), also on the stack, and the value of (h y) just returned.

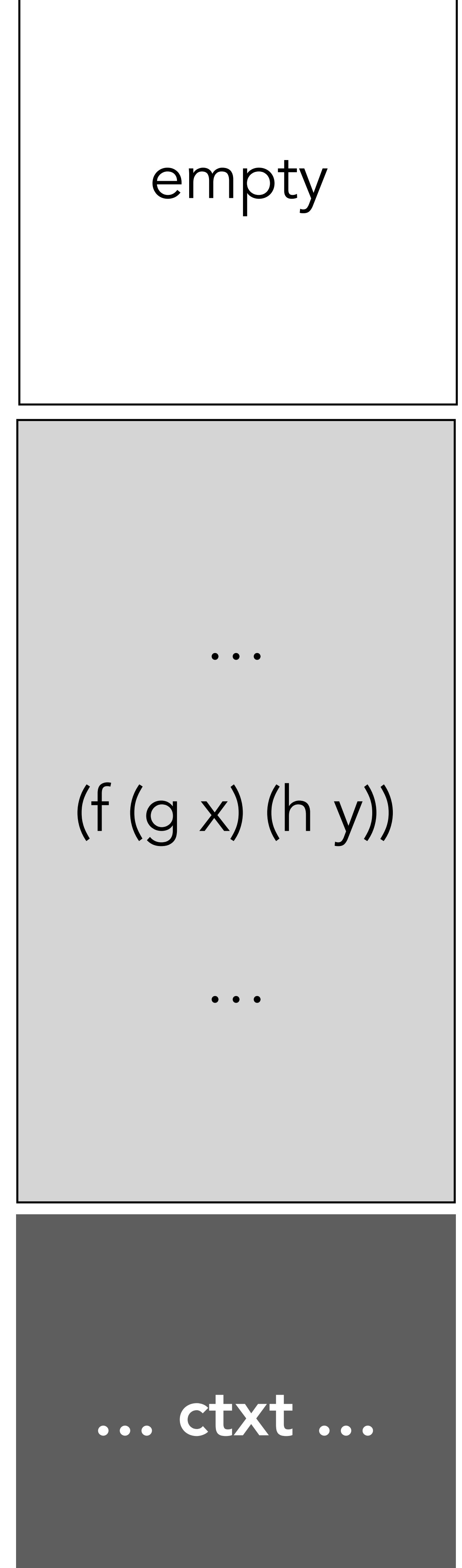


The stack

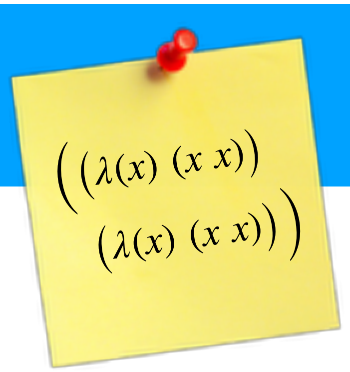
```
(f (g x) (h y))
```

This call may involve any number of pending expressions being saved and eliminated atop the stack.

When f returns, it’s value is returned to the original evaluation context.



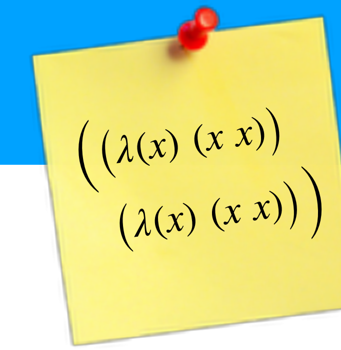
Example



For the following code, indicate when each subexpression is reached and returns.

```
(+ (f x) 1 (g (f y)))
```

Example



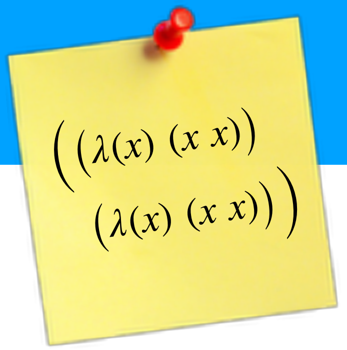
For the following code, indicate when each subexpression is reached and returns.

```
(+ (f x) 1 (g (f y)))
```

$$(+^{1}\ (f^{5}\ x^{6})^{2}\ 1^{3}\ (g^{7}\ (f^{9}\ y^{10})^{8})^{4})^{0}$$

With each subexpression labeled.

Example



For the following code, indicate when each subexpression is reached and returns.

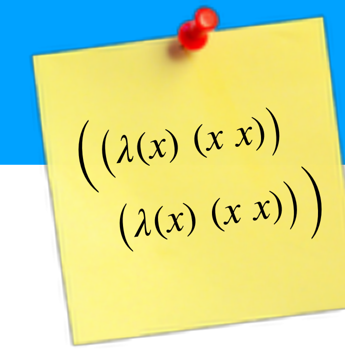
```
(+ (f x) 1 (g (f y)))
```

$$(+^{1}\ (\mathtt{f}^{5}\ \mathtt{x}^{6})^{2}\ 1^{3}\ (\mathtt{g}^{7}\ (\mathtt{f}^{9}\ \mathtt{y}^{10})^{8})^{4})^{0}$$

$$\llbracket^{0} \qquad \llbracket^{2}\ \llbracket\rrbracket^{5}\ \llbracket\rrbracket^{6}\ \mathsf{Call}^{f}\ \rrbracket^{2} \qquad \llbracket\rrbracket^{3}$$

$$\llbracket^{4}\ \llbracket\rrbracket^{7}\ \llbracket^{8}\ \llbracket\rrbracket^{9}\ \llbracket\rrbracket^{10}\ \mathsf{Call}^{f}\ \rrbracket^{8}\ \mathsf{Call}^{g}\ \rrbracket^{4} \quad \mathsf{Call}^{+}\ \rrbracket^{0}$$

Example



For the following code, indicate when each subexpression is reached and returns.

```
(let ([ls ‘(0)1])
  (if (null?6 ls7)3
      ‘()4
      (cons8 ls9 ls10)5)2)0
```

$$[\![^{0} \quad [\![]\!]^{1}\ \text{Bind}^{ls}\ [\![^{2}\ [\![^{3}\ [\![]\!]^{6}\ [\![]\!]^{7}\ \text{Call}^{null?}\ ]\!]^{3}$$

$$[\![^{5}\ [\![]\!]^{8}\ [\![]\!]^{9}\ [\![]\!]^{10}\ \text{Call}^{cons}\ ]\!]^{5} \quad]\!]^{2}\ ]\!]^{0}$$