Assignment 2
Introduction

CIS 352 — Spring 2020
Example

Consider that we have three rectangles, red, green, and blue

How much total area do they cover?
Observation: if we add areas, we get too much

This part counted twice
Inclusion Exclusion

- Include (add) the sum of their areas
- Exclude (subtract) the sum of their pairwise intersection
- Include (add) the sum of their triple-wise intersections
- Exclude (subtract) the sum of quadruple-wise intersections
- Keep going until each of the N-wise intersections has been accounted for...
Inclusion Exclusion

|A| + |B| + |C|

|A| + |B| + |C|
- (|A \cap B| + |A \cap C| + |B \cap C|)

|A| + |B| + |C|
- (|A \cap B| + |A \cap C| + |B \cap C|)
+ |A \cap B \cap C|
e2/e3/a2

• e2 (.25%+ bonus): rectangle operations
• e3 (.25%+ bonus): incl. excl. rectangle counting
• a2: same ideas as in e3, but can’t use incl. excl.
  • Algorithm must be $O(n \log(n))$
• Solution: use **QuadTree**
QuadTrees

• Generalization of binary trees
  • Each node has a point and four subtrees
  • Characterizes a rectangle!
• Each node rooted at a point in two-dimensional space
  • Four children characterize the rectangles to the…
    • Top right, bottom right, bottom left, and top left
Let’s say we wanted to characterize the rectangle from (0,0) to (5,2)…

Could use quadtree rooted at (0,0)
Quadtree rooted at a point $0,0$. 

\[
\left( \lambda(x) \ (x \ x) \ \right) \\
\left( \lambda(x) \ (x \ x) \ \right)
\]
To the top right of (0,0) is this giant space…

Which is characterized by this sub quadtree
To the top right of (0,0) is this giant space...

This means that the rectangle from (0,0) to (5,2) is covered.

Which is characterized by this sub quadtree.

Notice that it's bottom left is filled in green to indicate it is covered.
Can also think of this like a **decision diagram** on the space.
The TR of (0,0) is selecting all the space to the top right of (0,0).
Example

Which is already **cut off** at this boundary by virtue of having been nested inside the top-level quadtree.

Then, the **BR** of (5,2) **further** limits that to the **bottom right** of (5,2).
Note that the rest of the nodes are empty.

To the top left of (0,0) is empty space.
Think about how you could calculate the **area** of a quadtree.

Basic intuition: always keep track of the bottom left and top right coordinate.
Initially, bottom left coordinate covered is (-inf, -inf), and top right is (+inf, +inf)
Then, when we look at (0,0), we sum the areas of each of its children.
Then, when we look at (0,0), we sum the areas of each of its children. To calculate area of the top right of (0,0), we calculate it relative to the bottom left being at (0,0) rather than (-inf,-inf).
Then, when we look at (0,0), we sum the areas of each of its children.

Then, when we calculate the area of (5,2), we sum the area of each child.
Example

Then, when we look at (0,0), we sum the areas of each of its children

Think about how you could turn this idea into Racket code…

Including the covered one (empty has 0 area), which has area from (0,0) to (5,2)
Solving a2

- Figure out how to construct quad trees
  - Three constructors/cases: empty, covered, quad node
- In these slides, we discussed how to insert into empty
- You figure out how to do others
- Figure out how to calculate the total space of a quadtree
- Define a function \( \text{insert-rect } qt \ r \) (or any other name) that takes a quadtree and rectangle (to insert into it) and returns a new quadtree with that rectangle inserted
- Define a function that sums the total area of a quadtree
- Insert all rectangles to build quadtree, then get total area